H. -G M LLER, SQAD M LLER, F. AO

1. IN ROD C ION

(
1974),

(
1999),

(
1995),

(
1986),

(
1987),

(
1993),

(1998), (1998), (2000),

(2004),

g,

$$Y_j = g(t_j) + e_j(t_j), \qquad j = 1, \ldots, J.$$

$(t_j)_{j=1,\ldots,J}$ $v(t_j) =$ $(e_j(t_j))$

(X, Y),

$v(x) = E(Y^2 X = x) \quad E(Y X = x)^2$.

(2002, 2005).

(2002) (

1998).

1.

(2001)

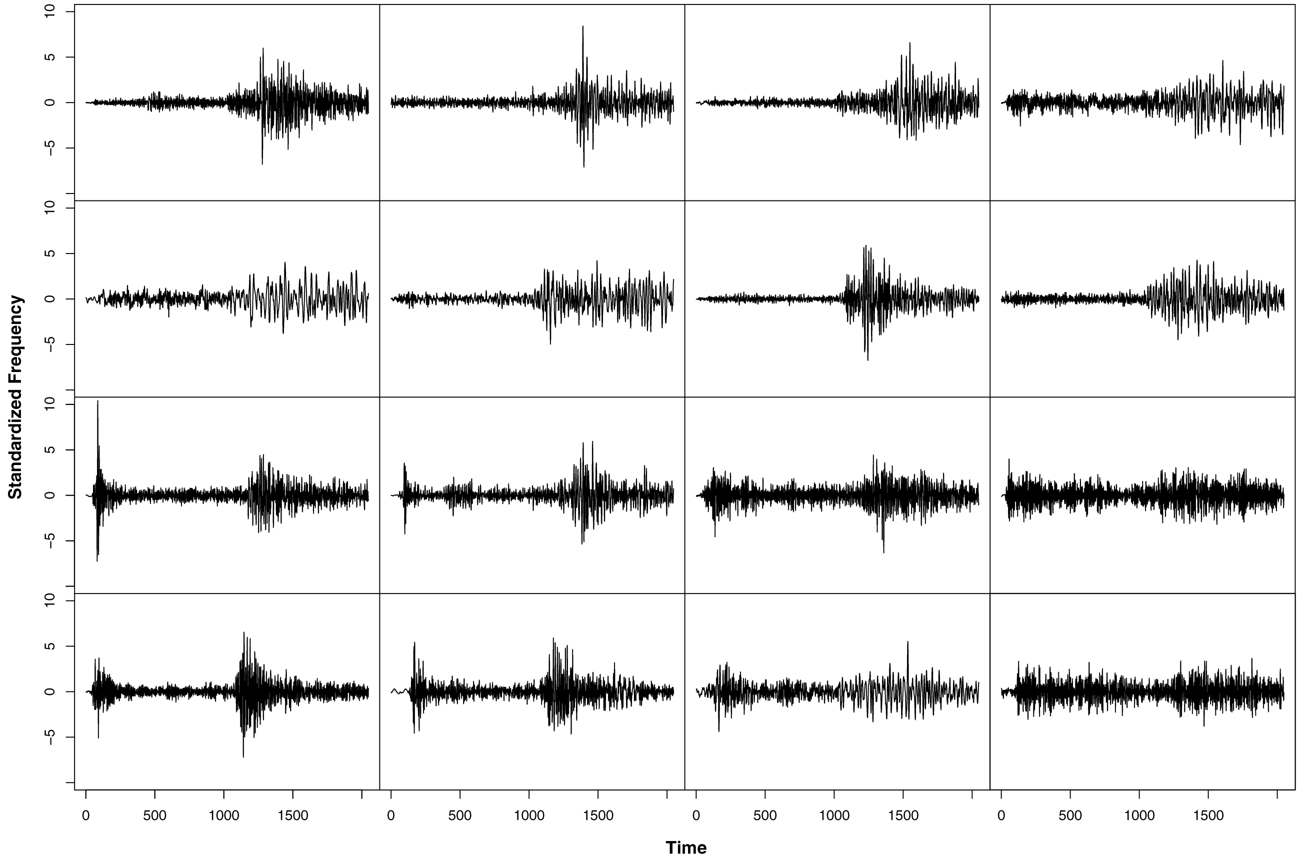
1

95616 (: *mueller@wald.ucdavis.edu*).

89069 (: *stamue@mathematik.uni-ulm.de*).

80523 (: *fyao@stat.colostate.edu*).

-03-54448 -05-05537.



F 1. D E), ( ), O E ( ). .025

1.

( )

1986).
(1950) (1958),
(1991;
1992; 2005;
2001; 2003).

2.

3,
4
1),
2),
3).

5,

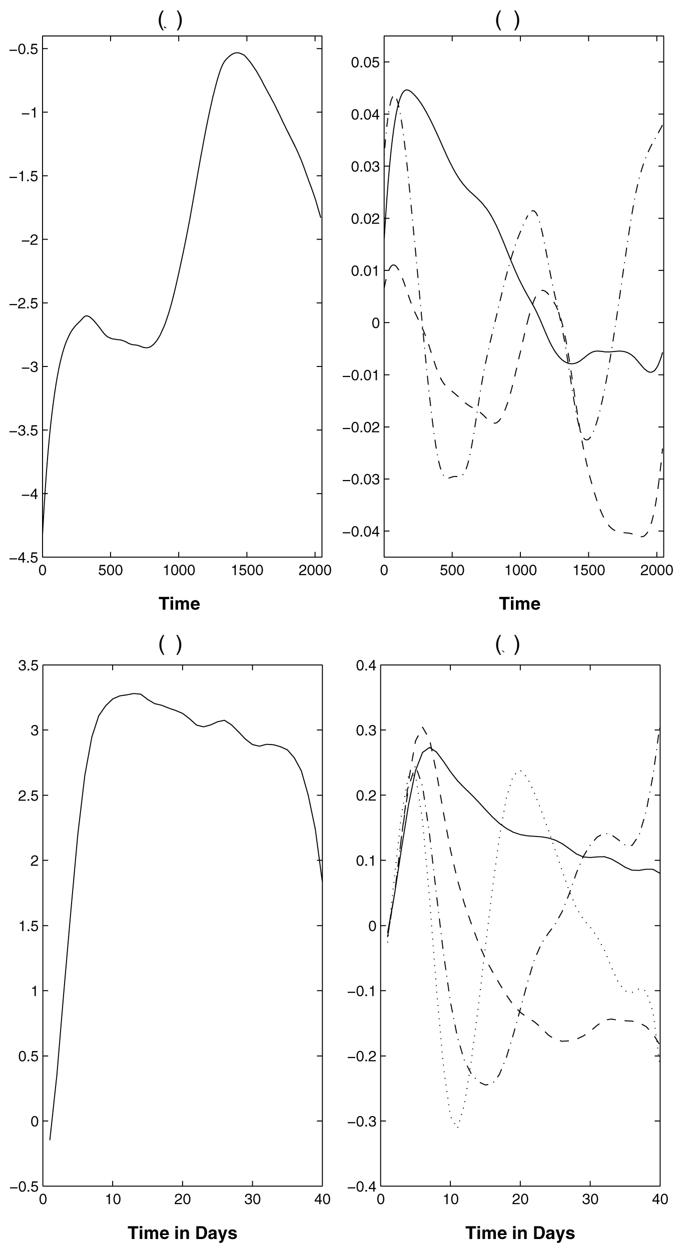
2. DECOMPOSING FUNCTIONAL DATA

S

V

S
n S_i,
$i = 1, \ldots, n.$ X_{ij}

R_{ij} i S $= a_1, a_2$ m (2.5). R_{ij},



F 2. C F P E E D ( ) ( ) E -L D ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ), (----), (- - - -) ( ) 62.8%, 23.6%, 7.8% ( ) ( ), (----), (- - - -), ( ) ( ) 48.3%, 21.0%, 11.6%, 6.7% .025 ( ) ( ), ( ) ( ).

t_{ij} $=$ a_1, a_2 S

$$X_{ij} = S_i(t_{ij}) + R_{ij}, \qquad i = 1, \ldots, n, j = 1, \ldots, m. \qquad (1)$$

R_{ij} R_{ij} $R_{i'k}$ $i \neq i'$,

$$ER_{ij} = 0, \qquad (R_{ij}) = \sigma^2_{Rij} < \infty.$$

(2)

$$(Z_{ij}, Z_{ik}) = \left(V_i(t_{ij}), V_i(t_{ik})\right) = G_V(t_{ij}, t_{ik}), \qquad j \neq k. \tag{9}$$

G_V Z_{ij}. σ_W^2. (1998) (2005). μ_V G_V (1991).

φ_k ψ_k, $k = 1, 2, \ldots,$ λ_k ρ_k $\sum_k \lambda_k < \infty$ $\sum_k \rho_k < \infty$. ξ_k $E(\xi_k) = 0$ $(\xi_k) = \lambda_k$ ζ_k $E(\zeta_k) = 0$ $(\zeta_k) = \rho_k$, μ_S μ_V

$$S(t) = \mu_S(t) + \sum_{k=1}^{\infty} \xi_k \varphi_k(t)$$
$$V(t) = \mu_V(t) + \sum_{k=1}^{\infty} \zeta_k \psi_k(t). \tag{10}$$

W_{ij} ε_{ij}, $i = 1, \ldots, n$, $j = 1, \ldots, m$, $E(W_{ij}) = 0$, $(W_{ij}) = \sigma_W^2$, $P(\varepsilon_{ij} > 0) = P(\varepsilon_{ij} < 0) = \frac{1}{2}$. X_{ij}

$$X_{ij} = S_i(t_{ij}) + (\varepsilon_{ij})\{ V_i(t_{ij}) + W_{ij} \}^{1/2}. \tag{11}$$

S V S, λ_k ρ_k 0 ξ_k ζ_k.

3. ESTIMATION OF MODEL COMPONENTS

Z_{ij} V_i (7), ζ_k V, (8). (2003), $V(t_{ij})$ Z_{ij} μ_V ψ_k μ_k ψ_k. $(W_{ij}) = \sigma_W^2$.

X_{ij} (t_{ij}), $i = 1, \ldots, n$, $j = 1, \ldots, m$, i j (1), (7)). (2005) (1998).

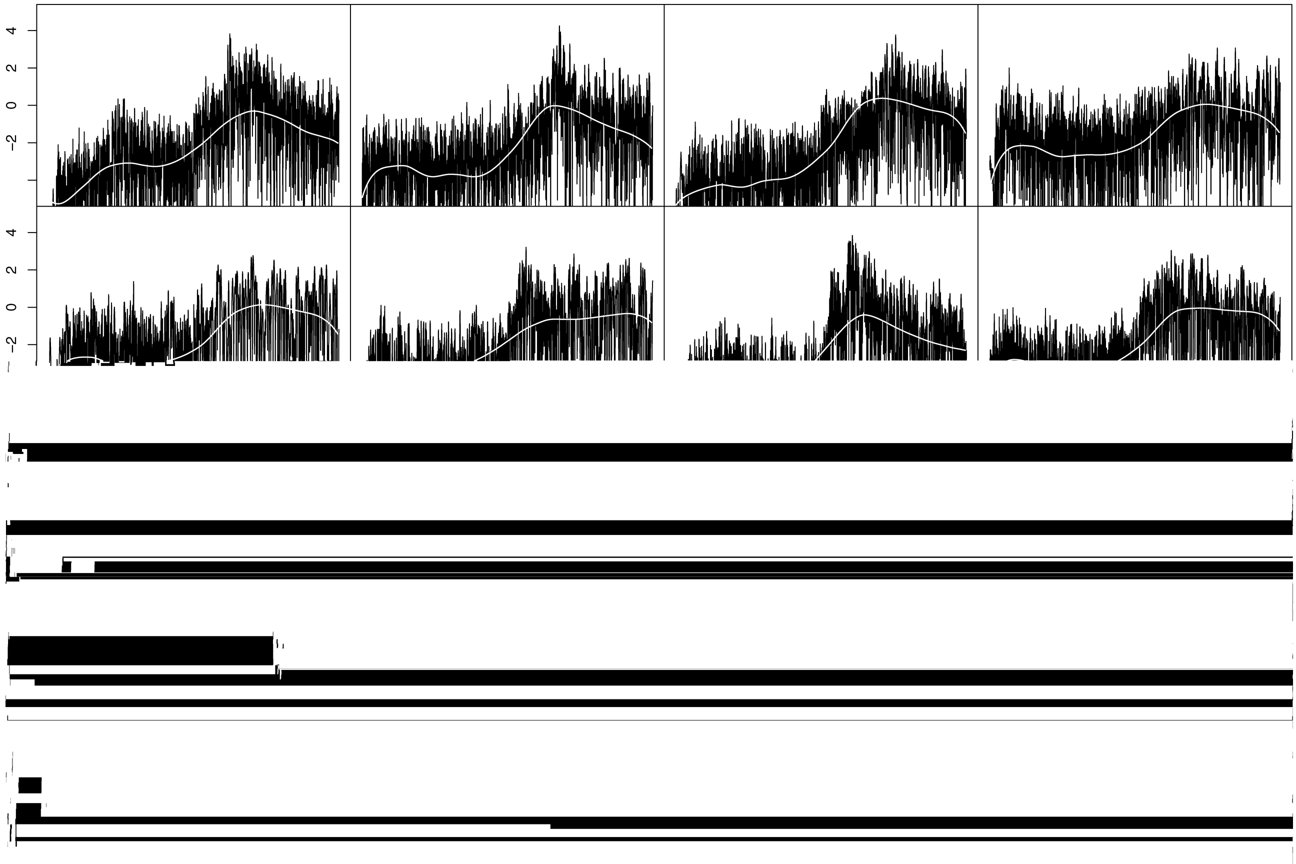
(t_{ij}, X_{ij}), $j = 1, \ldots, m$, S_i; $b_{S,i}$ $b_{S,i}$ S. $S_i(t_{ij})$ (.1) $S_i(t_{ij})$. $R_{ij} = X_{ij} \quad S_i(t_{ij})$.

$$Z_{ij} = (R_{ij}^2) = \left(X_{ij} \quad S_i(t_{ij})\right)^2, \qquad i = 1, \ldots, n, j = 1, \ldots, m. \tag{12}$$

Z_{ij}, $i = 1, \ldots, n$, $j = 1, \ldots, m$,

1. Z_{ij}, μ_V (4) b_V ; (.2) b_V
2. G_V (5) (.3) , h_V, $(t_{ij}, t_{ij'})$, $t_{ij} \neq t_{ij'}$, σ_W^2; (2).
3. ; (.4).
4. $(W_{ij}) = \sigma_W^2$ (2) b_{Q_W} σ_W^2 (.7). b_{Q_W}
5. ζ_j (8) (.5).

M V, ;



F 3. O = $(R^2(\ ))$ ( ). E F P ( ) (13) ( ) E E ( ), S E ( ). E ( ). ; F 1. .025

(.6). (2005), μ_V, ψ_k ρ_k, σ_W^2, ζ_{ik} (7), M

$$V_i(t) = \mu_V(t) + \sum_{k=1}^{M} \zeta_{ik}\psi_k(t). \qquad (13)$$

3.

4. ASYMPTOTIC RESULTS

(13) Z_{ij} (3) Z_{ij} (12) S_i b_S S_i (.1) ; b_S $b_{S,i}$,

$b_{S,i}$, b_S, (2.1), b_S, (2.2); b_V, h_V, b_{Q_V} μ_V (.2), $G_V(s,t)$ (.3), $Q_V(t)$ (.7) (2.3), (2.5).

m ( ) ( )

Theorem 1. (1), (2), (1.1), (2.1), $S_i(t)$

$$E\Big(_{t\in\mathcal{T}}\ S_i(t) \quad S_i(t)\Big) = O\Big(b_S^2 + \frac{1}{\sqrt{m}\,b_S}\Big). \qquad (14)$$

$\mu_V(t)$ $\mu_V(t)$, $G_V(s,t)$ $G_V(s,t)$, ρ_k $\psi_k(t)$ ρ_k ψ_k (.2), (.3), (.4) σ_W^2 (.7) σ_W^2 (2)

Theorem 2. (1), (8) (1.1), (2.2),

$$\sup_{t\in\mathcal{T}} |\hat\mu_V(t) - \mu_V(t)| = O_p\left(b_S^2 + \frac{1}{\sqrt{m}b_S} + \frac{1}{\sqrt{n}b_V}\right),$$

$$\sup_{s,t\in\mathcal{T}} |\hat G_V(s,t) - G_V(s,t)| = O_p\left(b_S^2 + \frac{1}{\sqrt{m}b_S} + \frac{1}{\sqrt{n}h_V^2}\right), \tag{15}$$

$$|\hat\sigma_W^2 - \sigma_W^2| = O_p\left(b_S^2 + \frac{1}{\sqrt{m}b_S} + \frac{1}{\sqrt{n}h_V^2} + \frac{1}{\sqrt{n}b_{Q_V}}\right).$$

ρ_k 1, ψ_k

$$\sup_{t\in\mathcal{T}} |\hat\psi_k(t) - \psi_k(t)| \xrightarrow{p} 0, \qquad \hat\rho_k \xrightarrow{p} \rho_k. \tag{16}$$

ρ_k ψ_k $\sup_{t\in\mathcal{T}} |\hat\psi_k(t) - \psi_k(t)| = O_p(\alpha_{nk} + \alpha_{nk}^*)$ $|\hat\rho_k - \rho_k| = O_p(\alpha_{nk} + \alpha_{nk}^*)$, α_{nk} α_{nk}^* (.1) (.1) (.2).

V_i (13) V_i, 3. ζ_{ik} (.5) ζ_{ik} V

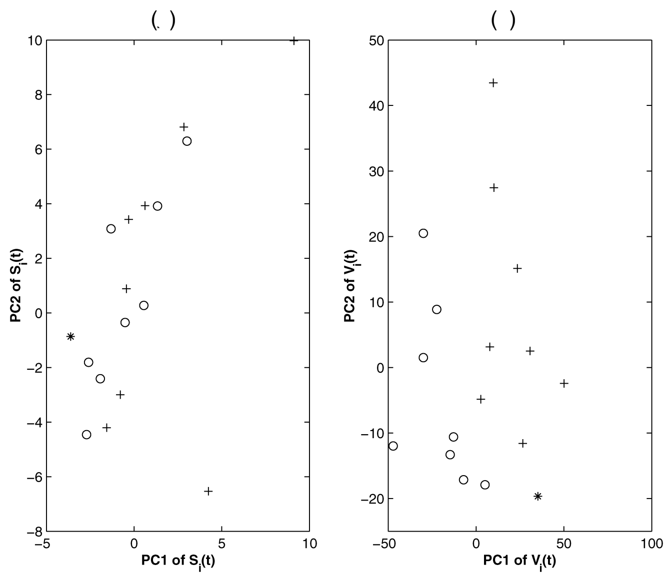
Theorem 3. (1), (8) (1.1), (2.2), V

$$\sup_{1\le k\le M} |\hat\zeta_{ik} - \zeta_{ik}| \xrightarrow{p} 0, \tag{17}$$

(13), $M = M(n) \to \infty$, $n \to \infty$. $V_i(t)$ V, $1 \le i \le n$,

$$\sup_{t\in\mathcal{T}} |\hat V_i(t) - V_i(t)| \xrightarrow{p} 0. \tag{18}$$

(16),



F 4. R … F E F P … C … PC2 … PC1, O … P … ( ), E … E … D (+, … ; … * …).

… $(X_{ij} - S_i(t_{ij}))^2$ … 0, … .001 … R_{ij}^2 …

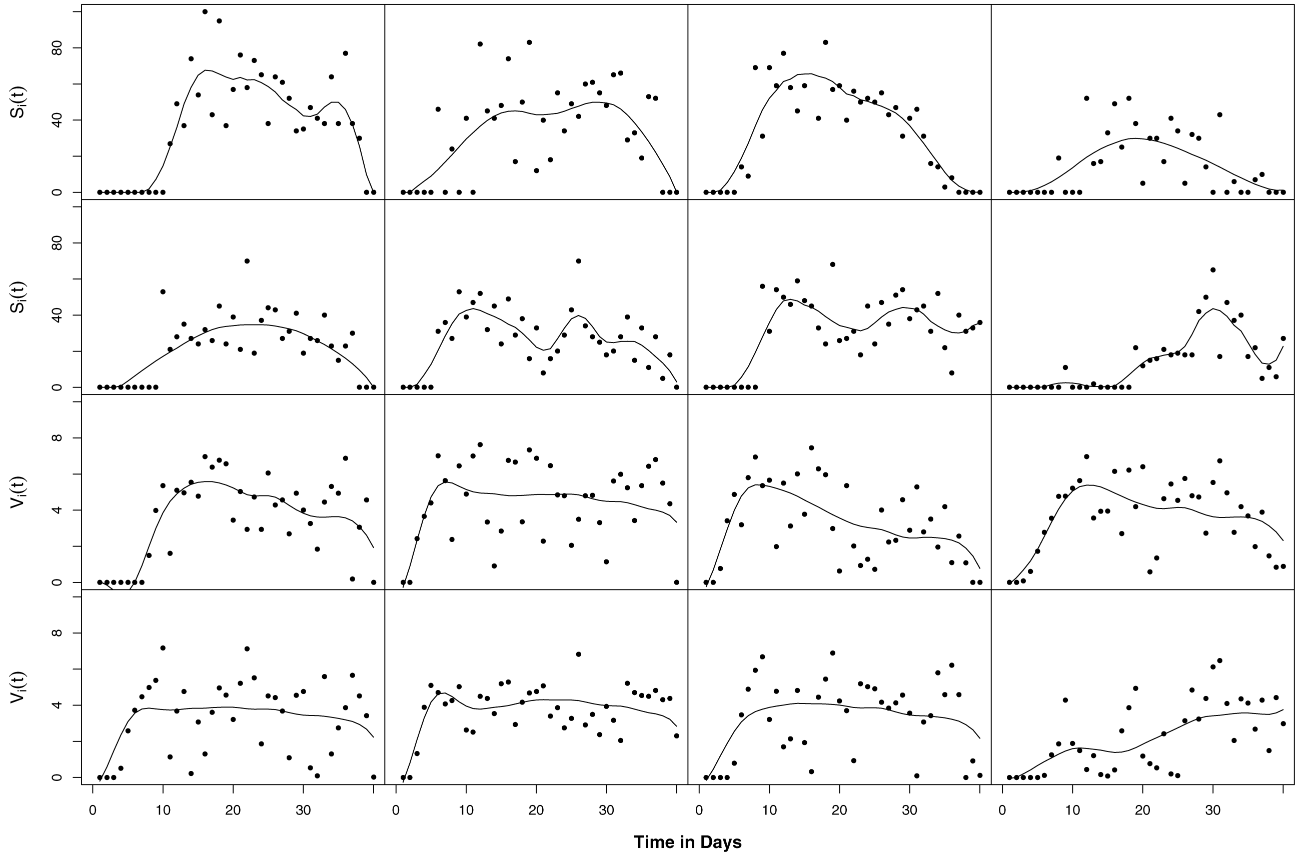
… V_i … 1 … 3. … 1. …

… ζ_{ik}, $k = 1, 2$, $i = 1, \ldots, 15$ … (.5) … (8) … V, … 4( ), … S (… 1991, … 2003) … 4( ). … V_i … S_i … V_i … S, … 4( ), … S, … (ζ_{i1}, ζ_{i2}) … S … 7 … (… 15 …), … 0 … S, …

… ζ_{ik}, $k = 1, 2$, … $V_i(t)$. …

5.2 E -L D

… 50.8(( )-410 7( )-0.2 …)8( )5 … 52



F[illegible] 5. O[illegible] E[illegible]-L[illegible] C[illegible] ( )[illegible] I[illegible] E[illegible] M[illegible] F[illegible] 40 D[illegible] A[illegible], B[illegible] C[illegible] L[illegible]-O[illegible]-O[illegible] C[illegible] E[illegible] ( ), O[illegible] = $(R^2(\,))(\,)$ E[illegible] R[illegible] (13) F[illegible] P[illegible] E[illegible] M[illegible] ( ).

[illegible]

APPENDI A: ES IMA ION PROCED RES

$\kappa_1(\cdot)$ $\kappa_2(\cdot,\cdot)$ (2.1) (2.2). $b_V = b_V(n)$ $h_V = h_V(n)$ μ_V (4) G_V (5) 1 2. Z_{ij}.

(1996). S_i, $i = 1, \dots, n$, (t_{ij}, X_{ij}), $j = 1, \dots, m$, $b_{S,i}$

$$\sum_{j=1}^{m} \kappa_1\left(\frac{t_{ij} \quad t}{b_{S,i}}\right)\{X_{ij} \quad \beta_{i,0} \quad \beta_{i,1}(t \quad t_{ij})\}^2 \qquad (.1)$$

$\beta_{i,0}$ $\beta_{i,1}$. $S_i(t_{ij}) = \beta_{i,0}(t_{ij})$. $b_{S,i}$ b_S (2.1). μ_V,

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \kappa_1\left(\frac{t_{ij} \quad t}{b_V}\right)\{Z_{ij} \quad \beta_0 \quad \beta_1(t \quad t_{ij})\}^2 \qquad (.2)$$

β_0 β_1, $\mu_V(t) = \beta_0(t)$. $_i(t_{ij_1}, t_{ij_2}) = (Z_i(t_{ij_1}) \quad \mu_V(t_{ij_1}))(Z_i(t_{ij_2}) \quad \mu_V(t_{ij_2}))$, $G_V(s,t)$

$$\sum_{i=1}^{n} \sum_{j_1 \neq j_2 \quad m} \kappa_2\left(\frac{t_{ij_1} \quad s}{h_V}, \frac{t_{ij_2} \quad t}{h_V}\right)$$

$$\{ \quad _i(t_{ij_1}, t_{ij_2}) \quad f(\beta, (s,t), (t_{ij_1}, t_{ij_2}))\}^2, \qquad (.3)$$

$f(\beta, (s,t), (t_{ij_1}, t_{ij_2})) = \beta_0 + \beta_{11}(s \quad t_{ij_1}) + \beta_{12}(t \quad t_{ij_2})$, $\beta = (\beta_0, \beta_{11}, \beta_{12})$, $G_V(s,t) = \beta_0(s,t)$.

$\{\rho_k, \psi_k\}_{k \quad 1}$ $\{\rho_k, \psi_k\}_{k \quad 1}$

$$\int_{\mathcal{T}} G_V(s,t)\psi_k(s)\,ds = \rho_k \psi_k(t), \qquad (.4)$$

$\{\psi_k\}_{k \quad 1}$ (2003). M ζ_{ik} (8),

$$\zeta_{ik} = \sum_{j=2}^{m} (Z_{ij} \quad \mu_V(t_{ij}))\psi_k(t_{ij})(t_{ij} \quad t_{i,j \quad 1}),$$

$$i = 1, \dots, n, k = 1, \dots, M. \qquad (.5)$$

$\mu_V^{(\quad i)}$ $\psi_k^{(\quad i)}$

$$V(M) = \sum_{i=1}^{n}\sum_{j=1}^{m}\{Z_{ij} \quad V_i^{(\quad i)}(t_{ij})\}^2, \qquad (.6)$$

M, $V_i^{(\quad i)}(t) = \mu_V^{(\quad i)}(t) + \sum_{k=1}^{M} \zeta_{ik}^{(\quad i)}\psi_k^{(\quad i)}(t)$ $\zeta_{ik}^{(\quad i)}$ (.5). V_i (13).

σ_W^2, G_V, $\tilde{G}_V(t)$ $\{G_V(t,t) + \sigma_W^2\}$ $Q_V(s)$, b_{Q_V}. $\mathcal{T} = a_1, a_2$, $\mathcal{T} = a_2 \quad a_1$ $\mathcal{T}_1 = a_1 + \mathcal{T}/4, a_2 \quad \mathcal{T}/4$

$$\sigma_W^2 = \frac{1}{\mathcal{T}_1}\int_{\mathcal{T}_1}\{Q_V(t) \quad \tilde{G}_V(t)\}_+\,dt \qquad (.7)$$

$\sigma_W^2 > 0$ $\sigma_W^2 = 0$ $\mathcal{T}/4$ (2003).

APPENDI B: AS MP IONS AND NO A IONS

S V

(1) $C > 0$ S V

$$_t \left|S^{(\nu)}(t)\right| < C \quad \nu = 0, 1, 2$$

$$_t \quad V(t) \quad < C.$$

$b_{S,i} = b_{S,i}(n)$, $b_V = b_V(n)$, $h_V = h_V(n)$, $b_{Q_V} = b_{Q_V}(n)$ S_i (.1), μ_V (.2), G_V (.3), $Q_V(t)$ (.7). n m

(2.1) $b_{S,i}$, b_S c_1 c_2, $0 < c_1 < \quad _i b_{S,i}/b_S \quad _i b_{S,i}/b_S < c_2 < \infty$.

(2.2) $m \to \infty$, $b_S \to 0$, $mb_S^2 \to \infty$.

(2.3) $b_V \to 0$, $b_{Q_V} \to 0$, $nb_V^4 \to \infty$, $nb_{Q_V}^4 \to \infty$, $_n n$ $b_V^6 < \infty$, $_n nb_{Q_V}^6 < \infty$.

(2.4) $h_V \to 0$, $nh_V^6 \to \infty$, $_n nh_V^8 < \infty$.

(2.5) $_n n^{1/2} b_V m \quad ^1 < \infty$, $_n n^{1/2} b_{Q_V} m \quad ^1 < \infty$, $_n n^{1/2} h_V m \quad ^1 < \infty$.

$\{t_{ij}\}_{i=1,\dots,n\; j=1,\dots,m}$ i $j = 1, \dots, m \quad 1$, $t_{ij} < t_{i,j+1}$ f $\int_{\mathcal{T}} f(t)\,dt = 1$ $_{t \in \mathcal{T}} f(t) > 0$. t_{ij} $t_{ij} = F \quad ^1(\frac{j \quad 1}{m \quad 1})$, $F \quad ^1$ $F(t) = \int_{a_1}^{t} f(s)\,ds$.

t_{ij} i, c_1 c_2 f_i $0 < c_1 < \quad _i \quad _{t \in \mathcal{T}} f_i(t) < \quad _i \quad _{t \in \mathcal{T}} f_i(t) < c_2$. N_i $m \to \infty$. c_1 c_2, $0 < c_1 < \quad _i \frac{N_i}{m} < \quad _i \frac{N_i}{m} < c_2 < \infty$; $N_i = m$, m

$n = \{t_{ij} \quad t_{i,j-1} : j = 2, \ldots, m\}$,

(3) $n = O(m^{-1})$, $n, m \to \infty$.

X_{ij} Z_{ij} $t \in \mathcal{T}$,

(4) $_j E X_{ij}^4 < \infty$ $_j E Z_{ij}^4 < \infty$.

(1989). $(f \quad g)(h) = \langle f, h \rangle g$ $f, g, h \in H$, H F $\sigma_2(H)$, $\langle T_1, T_2 \rangle_F = (T_1 T_2^*) = \sum_j \langle T_1 u_j, T_2 u_j \rangle_H$ $T_F^2 = \langle T, T \rangle_F$, $T_1, T_2, T \in F$, T_2^* T_2, $\{u_j : j \quad 1\}$ H. $\mathbf{G}_V$ (6) $\mathbf{G}_V$, G_V (5), G_V (.3), $\mathcal{I}_i = \{j : \rho_j = \rho_i\}$ $\mathcal{I}' = \{i : \mathcal{I}_i = 1\}$, $\mathcal{I}_i$ $\mathcal{I}_i$. $\mathbf{P}_j^V = \sum_{k \in \mathcal{I}_j} \psi_k \quad \psi_k$ $\mathbf{P}_j^V = \sum_{k \in \mathcal{I}_j} \psi_k \quad \psi_k$ H. $\{\psi_k : k \in \mathcal{I}_j\}$ $\{\psi_k : k \in \mathcal{I}_j\}$. j,

$$\delta_j^V = \frac{1}{2} \{ \rho_l \quad \rho_j : l \notin \mathcal{I}_j\}, \qquad (.1)$$

$\Lambda_{\delta_j^V} = \{z \in \mathcal{C} : z \quad \rho_j = \delta_j^V\}$, $\mathcal{C}$ $\mathbf{G}_V$ $\mathbf{G}_V$ $\mathbf{R}_V$ $\mathbf{R}_V$, $\mathbf{R}_V(z) = (\mathbf{G}_V \quad zI)^{-1}$ $\mathbf{R}_V(z) = (\mathbf{G}_V \quad zI)^{-1}$.

$$A_{\delta_j^V} = \left\{ \mathbf{R}_V(z)_F : z \in \Lambda_{\delta_j^V} \right\} \qquad (.2)$$

$M = M(n)$ $V(t)$, $V_i(t) = \mu_V(t) + \sum_{m=1}^{M(n)} \zeta_{im} \psi_m(t)$ (13). $\pi_\infty =$ $_{t \in \mathcal{T}} \pi(t)$ $\pi(\,)$ $\mathcal{T}$. μ_V ψ_j $M = M(n)$ n m $n \to \infty$,

(5) $\tau_n = \sum_{j=1}^{M} (\delta_j^V A_{\delta_j^V} \psi_{j\infty}) / (\bar{n} h_V^2 \quad A_{\delta_j^V}) \to 0$, $M = M(n) \to \infty$;

(6) $\sum_{j=1}^{M} \psi_{j\infty} = o(\{\bar{n} b_V, \bar{m}\})$ $\sum_{j=1}^{M} \psi_{j\infty} \psi'_{j\infty} = o(m)$.

(5) (6) ψ_j j $\theta_m = b_S^2 + (\bar{m} b_S)^{-1}$, V :

(7) $E\{ _{t \in \mathcal{T}} V(t) \quad V^{(M)}(t)^2\} = o(n)$, $V^{(M)}(t) = \mu_V(t) + \sum_{k=1}^{M} \zeta_k \psi_k(t)$.

(8) $1 \quad i \quad n$, $\theta_m \sum_{k=1}^{M} \psi_{k\infty}^2 = o_p(1)$ $\gamma_n = \sum_{k=1}^{M} (\delta_k^V A_{\delta_k^V} \psi_{k\infty}) / (\theta_m^{-1} \quad A_{\delta_k^V}) \to 0$ $n \to \infty$.

$t = t_{ij}, t_1 = t_{ij_1}$, $t_2 = t_{ij_2}$, i, j, j_1, j_2, $g(x \; t)$ X_{ij} $g_2(x_1, x_2 \; t_1, t_2)$ (X_{ij_1}, X_{ij_2}). $f(z \; t)$ $f_2(z_1, z_2 \; t_1, t_2)$ Z_{ij} (Z_{ij_1}, Z_{ij_2}). $g(\;t), f(\;t), t \in \mathcal{T}$, $g_2(\;t_1, t_2), f_2(\;t_1, t_2), t_1, t_2 \in \mathcal{T}$, :

(1.1) $(d^2/dt^2) g(x \; t)$ $(d^2/dt^2) f(z \; t)$ $\Re$ $\mathcal{T}$.

(1.2) $(d^2/dt_1{}^1 dt_2{}^2) g_2(x_1, x_2 \; t_1, t_2)$ $(d^2/dt_1{}^1 dt_2{}^2) f_2(z_1, z_2 \; t_1, t_2)$ $\Re^2$ $\mathcal{T}^2$, $_1 + _2 = 2, 0 \quad _1, _2 \quad 2$.

$\kappa_1 : \Re \to \Re$ $\kappa_2 : \Re^2 \to \Re$ $\kappa_1(u)$ $\kappa_2(u, v)$ $\chi_1(t) = \int e^{-iut} \kappa_1(u)\, du$ $\chi_2(t, s) = \int e^{-(iut+ivs)} \kappa_2(u, v)\, du\, dv$. :

(2.1) κ_1 $\kappa_1{}^2 = \int \kappa_1^2(u)\, du < \infty$, $\chi_1(t)$ $\int \chi_1(t)\, dt < \infty$.

(2.2) κ_2 $\kappa_2{}^2 = \int\int \kappa_2^2(u, v)\, du\, dv < \infty$, κ_2 0 u v, $\chi_2(t, s)$ $\int\int \chi_2(t, s)\, dt\, ds < \infty$.

APPENDI C: PROOFS

P m 1

W S V, $= _1 \quad _2$ $\overset{*}{E}$ $_2$ (S V), $\omega_1 \in _1$ ω_1, $R_j = R_j(\omega_1)$ i R_{ij} (1) $_2$ j, $\overset{*}{E}(R_j) = 0$ $_j \overset{*}{E}(R_j^2) < C_1$ C_1 (1) $_3$ $_2$ (1979).

$$\overset{*}{E}\left(_{t \in \mathcal{T}} S(t) \quad S(t) \right)(\omega_1) = O\left(b_S^2 + \frac{1}{\bar{m} b_S}\right)(\omega_1).$$

ω_1, $S^{(v)}(\omega_1)$, $v = 0, 1, 2$, $V(\omega_1)$ (1), ω_1;

$$_{\omega_1 \in _1} \overset{*}{E}\left(_{t \in \mathcal{T}} S(t) \quad S(t) \right)(\omega_1) = O\left(b_S^2 + \frac{1}{\bar{m} b_S}\right),$$

(14).

.1. V $\{t_{ij}, Z_{ij}\}$ ( ) $\mu_V, G_V, \sigma_W^2, \psi_k, \rho_k$, ζ_{ik}, (.2), (.3), (.4), (.5)

$$\alpha_{nk} = \frac{\delta_k^V A_{\delta_k^V}}{\bar{n} h_V^2 \quad A_{\delta_k^V}} \qquad \overset{*}{\alpha}_{nk} = \frac{\delta_k^V A_{\delta_k^V}}{\theta_m^{-1} \quad A_{\delta_k^V}}, \qquad (.1)$$

$\theta_m = b_S^2 + (\bar{m} b_S)^{-1}$ δ_k^V $A_{\delta_k^V}$ (.1) (.2).

Lemma C.1. (2.1), (2.3), (3), (5), (1.1), (2.2),

$$_{t \in \mathcal{T}} \mu_V(t) \quad \mu_V(t) = O_p\left(\frac{1}{\bar{n} b_V}\right)$$

$$_{s,t \in \mathcal{T}} G_V(s, t) \quad G_V(s, t) = O_p\left(\frac{1}{\bar{n} h_V^2}\right). \qquad (.2)$$

[illegible] ρ_k [illegible] 1, ψ_k [illegible]

$$\sup_{t\in\mathcal{T}} |\hat{\psi}_k(t) - \psi_k(t)| = O_p(\alpha_{nk}) \quad \text{[illegible]}$$
$$|\hat{\rho}_k - \rho_k| = O_p(\alpha_{nk}), \tag{C.3}$$

[illegible] $\alpha_{nk} \to 0$ [illegible] $n \to \infty$, k [illegible], α_{nk} [illegible] (C.1), [illegible] $O_p(\,)$ [illegible] (C.3) [illegible] 1 [illegible] k [illegible] M. [illegible] (C.2),

$$\sup_{t\in\mathcal{T}} |\hat{\sigma}_W^2(t) - \sigma_W^2(t)| = O_p\left([illegible] \left\{ \frac{1}{\sqrt{n}h_V^2}, \frac{1}{\sqrt{n}b_{Q_V}} \right\} \right). \tag{C.4}$$

[illegible] (1), (7) [illegible] (1.1), (2.2),

$$\sup_{1\le k\le M} |\hat{\zeta}_{ik} - \zeta_{ik}| \xrightarrow{p} 0 \quad \text{[illegible]}$$
$$\sup_{t\in\mathcal{T}} \left| \sum_{k=1}^{M} \hat{\zeta}_{ik}\hat{\psi}_k(t) - \sum_{k=1}^{\infty} \zeta_{ik}\psi_k(t) \right| \xrightarrow{p} 0, \tag{C.5}$$

[illegible] M [illegible] $M = M(n) \to \infty$ [illegible] $n \to \infty$.

Proof of Lemma C.1. [illegible] (C.2), (C.3), [illegible] (C.5) [illegible] 1 [illegible] 1 [illegible] (2006). [illegible] σ_W^2

$$+ \sup_{t\in\mathcal{T}} \left\{ \left| \sum_{k=1}^{M} \zeta_{ik}\psi_k(t) \quad \sum_{k=1}^{\infty} \zeta_{ik}\psi_k(t) \right| \right\}$$

$$Q_{i1}(n) + Q_{i2}(n),$$

$Q_{i1}(n) \xrightarrow{p} 0$ $Q_{i2}(n) \xrightarrow{p} 0.$

(.5), $Q_{i2}(n) \xrightarrow{p} 0$ (1), (7),

$Q_{i2}(n) = O(\vartheta_{in}^{(2)})$, $O(\,)$

1 i n. $Q_{i1}(n)$,

$$Q_{i1}(n) \quad \sup_{t\in\mathcal{T}} \left\{ \sum_{k=1}^{M} \zeta_{ik} \quad \zeta_{ik} \quad \psi_k(t) \right.$$

$$\left. + \sum_{k=1}^{M} \zeta_{ik} \quad \psi_k(t) \quad \psi_k(t) \right\}. \quad (.11)$$

(.10), (.11)

$$C_1\theta_{im} \sum_{k=1}^{M} \psi_k \,{}^2_\infty + \gamma_n \left\{ C_2 + \sum_{j=2}^{m} Z_{ij}\,(t_{ij} \quad t_{i,j} \quad 1) \right\} \xrightarrow{p} 0.$$

(.11) $O_p\{\sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} E\,\zeta_{ik}\,/(\theta_m{}^1 \quad A_{\delta_k^V})\}$. $E\{\sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} E\,\zeta_{ik}\,/(\theta_m{}^1 \quad A_{\delta_k^V})\}$ $\sum_{k=1}^{M} \delta_k^V A_{\delta_k^V}\,\overline{\rho_k}/(\theta_m{}^1 \quad A_{\delta_k^V})$ γ_n, $\rho_k \to 0$, 0 $Q_{i1}(n) = O_p(\vartheta_{in}^{(1)})$, $O_p(\,)$ i. (18) $\sup_{t\in\mathcal{T}} V_i(t) \quad V_i(t) = O_p(\vartheta_{in}^{(1)} + \vartheta_{in}^{(2)})$, $O_p(\,)$ 1 i n.

[Received December 2004. Revised December 2005.]

REFERENCES

(1998), *Journal of Gerontology–Biological Sciences*, 53, 245 251.

(1986), *Technometrics*, 28, 329 337.

(1999), *The Annals of Statistics*, 27, 36 64.

(1989), *Methods of Mathematical Physics*,

(1998), *Journal of the Royal Statistical Society*, , 60, 693 708.

(1993), *Journal of the Royal Statistical Society*, , 55, 145 155.

(1996), *Local Polynomial Modelling and Its Applications*, &

(1998), *Biometrika*, 85, 645 660.

(1986), *Biometrika*, 73, 625 633.

(1950), *Arkiv för Matematik*, 195 276.

(2001), *Technometrics*, 43, 1 9.

(2001), *Biometrika*, 87, 587 602.

(1992), *The American Statistician*, 46, 140 145.

(1998), *Journal of the American Statistical Association*, 441, 328 340.

(1987), *The Annals of Statistics*, 15, 610 625.

(1995), *The Annals of Statistics*, 23, 946 967.

(2002), *Applied Functional Data Analysis*,

——— (2005), *Functional Data Analysis*,

(1958), *Biometrics*, 14, 1 17.

(1991), *Journal of the Royal Statistical Society*, , 53, 233 243.

(1979), *The Annals of Statistics*, 7, 139 149.

(2002), *Computing Science and Statistics—Geoscience and Remote Sensing, Proceedings of the Interface*, 34, 373 379.

(1998), *Journal of the American Statistical Association*, 93, 1403 1418.

(1974), *Biometrika*, 61, 439 447.

(2006), *Journal of the Royal Statistical Society*, , 68, 3 25.

(2003), 6 .1( )-2285.56 19 -37 3776.4( )2.5( )8( ).