H. -G M LLER, AD M LLER, F. AO

1. IN ROD C ION

1974), (1999), 1995), (1986), 1987), 1993). (1998), (1998), (2000), (2004), g,

$$Y_j = g(t_j) + e_j(t_j), \qquad j = 1, \ldots, J.$$

$(t_j)_{j=1,\ldots,J}$ $v(t_j) =$ $(e_j(t_j))$

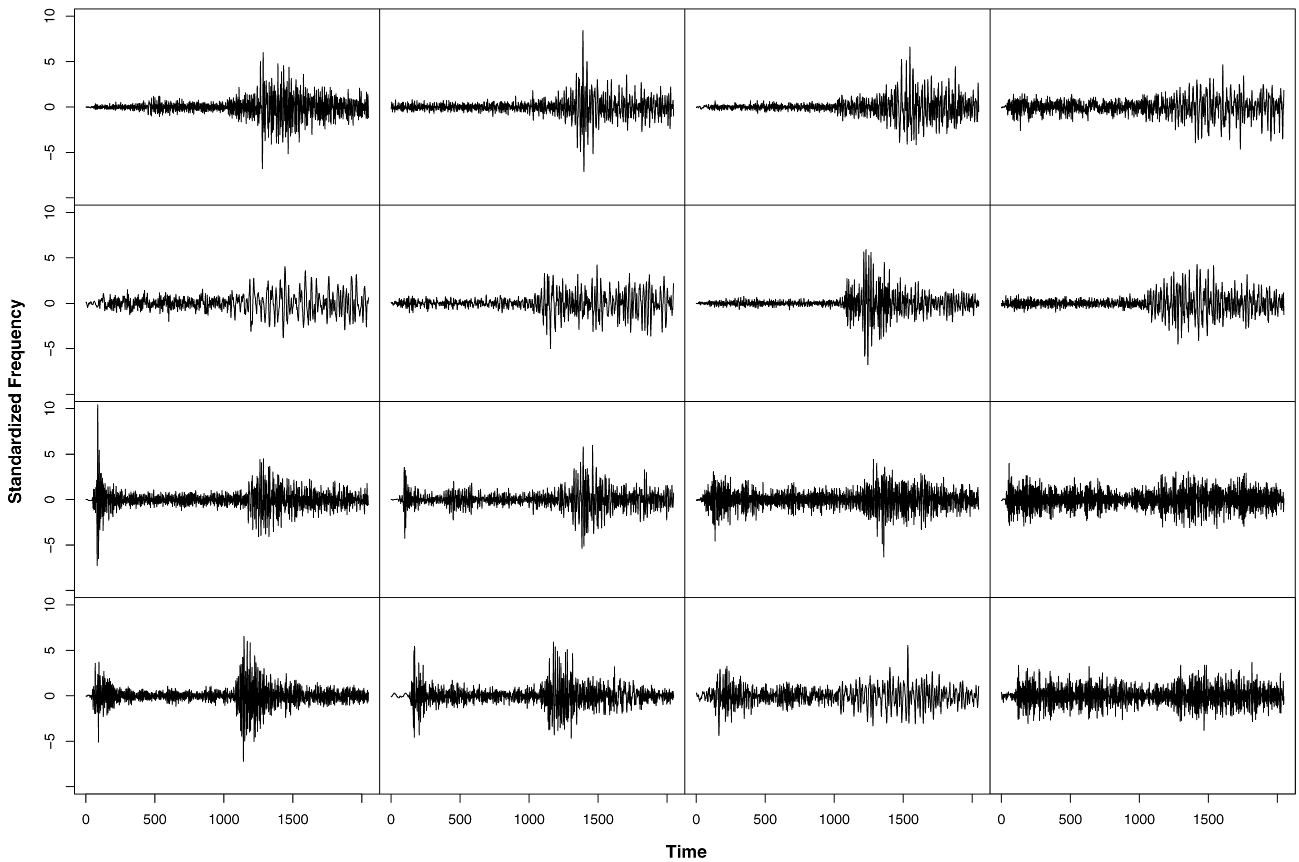
(X, Y),

$$v(x) = E(Y^2|X = x) - [E(Y|X = x)]^2.$$

(2002, 2005). (2002) (1998). 1. (2001) 1

95616 (: mueller@wald.ucdavis.edu). 89069 (: stamue@mathematik.uni-ulm.de). 80523 (: fyao@stat.colostate.edu).

-03-54448 -05-05537.

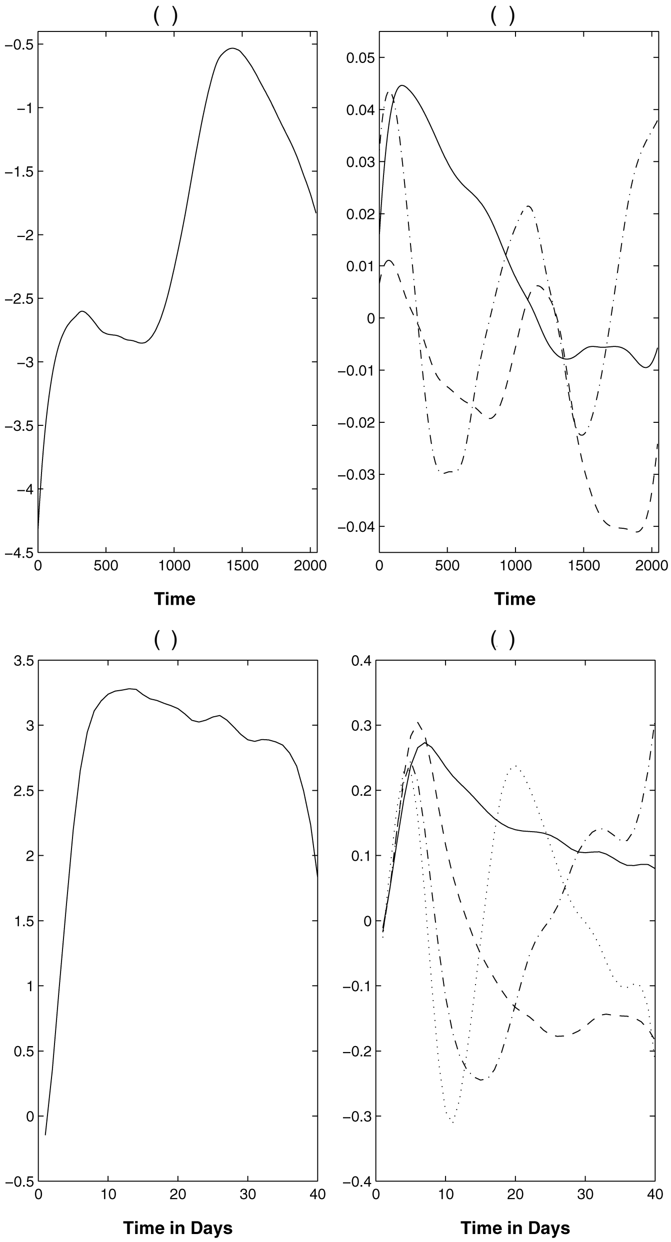


F 1. D E ( ), E ( ), O. E ( ). .025

1986).
(1950) (1958),
(1991;
1992; 2005;
2001; . 2003).

2. DECOMPOSING F NC IONAL DA A

$i = 1, \ldots, n$. X_{ij}



F 2. C E E D ( ) ( ) E -L D ( ) ( ) ( ) ( ) Ǫ ( ) ( ) ( ). ( ) Ǫ ( ), (----), (-·-·-·-) ( ) 62.8%, 23.6%, 7.8% ( ) Ǫ ( ), (----), (-·-·-·-), (······) ( ) 48.3%, 21.0%, 11.6%, 6.7% .025 ( ) ( ), ( ) ( ).

t_{ij} $T = [a_1, a_2]$ S

$$X_{ij} = S_i(t_{ij}) + R_{ij}, \qquad i = 1, \ldots, n, j = 1, \ldots, m. \tag{1}$$

R_{ij} , R_{ij} $R_{i'k}$ $i \neq i'$,

$$ER_{ij} = 0, \qquad (R_{ij}) = \sigma^2_{Rij} < \infty.$$

R_{ij} i S $T = [a_1, a_2]$. m (2.5). R_{ij}, -367.6(28.10 /

, (2)

$$(Z_{ij}, Z_{ik}) = \big(V_i(t_{ij}), V_i(t_{ik})\big) = G_V(t_{ij}, t_{ik}), \qquad j \neq k. \tag{9}$$

G_V Z_{ij}. σ_W^2. (1998) (2005). μ_V G_V (1991).

T, φ_k ψ_k, $k = 1, 2, \ldots$, λ_k ρ_k $\sum_k \lambda_k < \infty$ $\sum_k \rho_k < \infty$. ξ_k $E(\xi_k) = 0$ $(\xi_k) = \lambda_k$ ζ_k $E(\zeta_k) = 0$ $(\zeta_k) = \rho_k$, μ_S μ_V T,

$$S(t) = \mu_S(t) + \sum_{k=1}^{\infty} \xi_k \varphi_k(t)$$
$$V(t) = \mu_V(t) + \sum_{k=1}^{\infty} \zeta_k \psi_k(t). \tag{10}$$

W_{ij} ε_{ij}, $i = 1, \ldots, n$, $j = 1, \ldots, m$, $E(W_{ij}) = 0$, $(W_{ij}) = \sigma_W^2$, $P(\varepsilon_{ij} > 0) = P(\varepsilon_{ij} < 0) = \frac{1}{2}$. X_{ij}

$$X_{ij} = S_i(t_{ij}) + (\varepsilon_{ij})\{[V_i(t_{ij}) + W_{ij}]\}^{1/2}. \tag{11}$$

S V S, λ_k ρ_k 0 ξ_k ζ_k.

3. ESTIMATION OF MODEL COMPONENTS

Z_{ij} V_i (7), ζ_k V, (8). (2003), $V(t_{ij})$ $\hat{Z}_{ij}$ μ_V ψ_k $\hat{\mu}_k$ $\hat{\psi}_k$. $(W_{ij}) = \sigma_W^2$.

X_{ij} (t_{ij}), $i = 1, \ldots, n$, $j = 1, \ldots, m$, i j (1), (7) ( ). (2005) (1998).

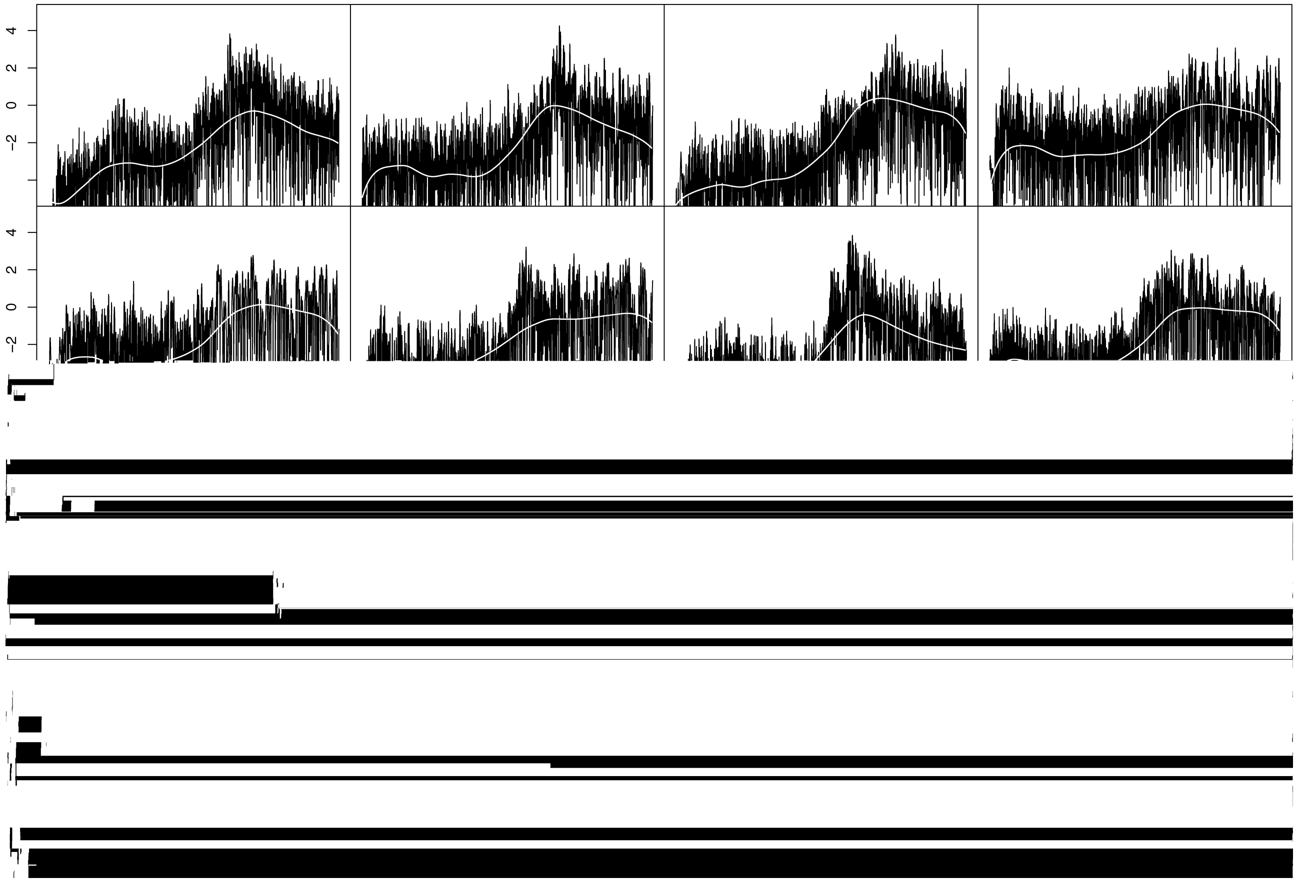
; (t_{ij}, X_{ij}), $j = 1, \ldots, m$, S_i; $b_{S,i}$; $b_{S,i}$ S. $\hat{S}_i(t_{ij})$ (.1) $S_i(t_{ij})$. $\hat{R}_{ij} = X_{ij} - \hat{S}_i(t_{ij})$

$$\hat{Z}_{ij} = (\hat{R}_{ij}^2) = \big(X_{ij} - \hat{S}_i(t_{ij})\big)^2, \quad i = 1, \ldots, n, j = 1, \ldots, m. \tag{12}$$

$\hat{Z}_{ij}$, $i = 1, \ldots, n$, $j = 1, \ldots, m$, :

1. $\hat{Z}_{ij}$, μ_V (4) b_V; (.2) b_V
2. G_V (5) (.3), h_V, $(t_{ij}, t_{ij'})$, $t_{ij} \neq t_{ij'}$, σ_W^2; (2).
3. ; (.4).
4. $(W_{ij}) = \sigma_W^2$ (2). b_{Q_W} $\hat{\sigma}_W^2$ (.7). b_{Q_W}
5. ζ_j (8) (.5).

M V, ;



F 3. O $= (R^2(\,))$ ( ). E F P $\hat{}()$ (13) ( ) E E ( ), Q E ( ), E ( ). ; F 1. .025 .

(.6). (2005), $\hat{\mu}_V$, $\hat{\psi}_k$ $\hat{\rho}_k$, $\hat{\sigma}_W^2$, $\hat{\zeta}_{ik}$. (7), M

$$\hat{V}_i(t) = \hat{\mu}_V(t) + \sum_{k=1}^{M} \hat{\zeta}_{ik}\hat{\psi}_k(t). \qquad (13)$$

3.

4. A Q MP O IC RE Q L Q

(13) Z_{ij} (3) $\hat{Z}_{ij}$ (12) S_i. b_S S_i (.1) ; b_S $b_{S,i}$,

$b_{S,i}$, b_S, (2.1), b_S, (2.2); b_V, h_V, b_{Q_V} $\hat{\mu}_V$ (.2), $\hat{G}_V(s,t)$ (.3), $\hat{Q}_V(t)$ (.7) (2.3), (2.5).

m ( ) ( ).

Theorem 1. (1), (2), (1.1), (2.1), $\hat{S}_i(t)$

$$E\Big(\sup_{t\in\mathcal{T}} |\hat{S}_i(t) - S_i(t)|\Big) = O\Big(b_S^2 + \frac{1}{\sqrt{m b_S}}\Big). \qquad (14)$$

$\hat{\mu}_V(t)$ $\mu_V(t)$, $\hat{G}_V(s,t)$ $G_V(s,t)$, $\hat{\rho}_k$ $\hat{\psi}_k(t)$ ρ_k ψ_k. (.2), (.3), (.4) $\hat{\sigma}^2_W$ (.7) σ^2_W (2).

Theorem 2. (1), (8) (1.1), (2.2),

$$\sup_{t\in\mathcal{T}} |\hat{\mu}_V(t) - \mu_V(t)| = O_p\left(b_S^2 + \frac{1}{\sqrt{m}b_S} + \frac{1}{\sqrt{n}b_V}\right),$$

$$\sup_{s,t\in\mathcal{T}} |\hat{G}_V(s,t) - G_V(s,t)| = O_p\left(b_S^2 + \frac{1}{\sqrt{m}b_S} + \frac{1}{\sqrt{n}h_V^2}\right), \tag{15}$$

$$|\hat{\sigma}^2_W - \sigma^2_W| = O_p\left(b_S^2 + \frac{1}{\sqrt{m}b_S} + \frac{1}{\sqrt{n}h_V^2} + \frac{1}{\sqrt{n}b_{Q_V}}\right).$$

ρ_k 1, $\hat{\psi}_k$

$$\sup_{t\in\mathcal{T}} |\hat{\psi}_k(t) - \psi_k(t)| \xrightarrow{p} 0, \qquad \hat{\rho}_k \xrightarrow{p} \rho_k. \tag{16}$$

$\hat{\rho}_k$ $\hat{\psi}_k$ $\sup_{t\in\mathcal{T}} |\hat{\psi}_k(t) - \psi_k(t)| = O_p(\alpha_{nk} + \alpha^*_{nk})$ $|\hat{\rho}_k - \rho_k| = O_p(\alpha_{nk} + \alpha^*_{nk})$, α_{nk} α^*_{nk} (.1) (.1) (.2).

$\hat{V}_i$ (13) V_i, 3. $\hat{\zeta}_{ik}$ (.5) ζ_{ik} V.

Theorem 3. (1), (8) (1.1), (2.2), V

$$\sup_{1\le k\le M} |\hat{\zeta}_{ik} - \zeta_{ik}| \xrightarrow{p} 0, \tag{17}$$

(13), $M = M(n) \to \infty$, $n \to \infty$, $\hat{V}_i(t)$ V, $1 \le i \le n$,

$$\sup_{t\in\mathcal{T}} |\hat{V}_i(t) - V_i(t)| \xrightarrow{p} 0. \tag{18}$$

(16),

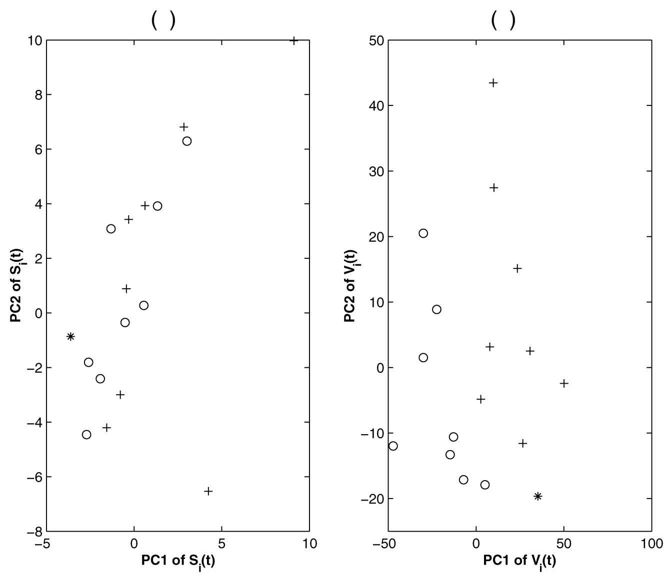


Figure 4. R … PC2 … PC1, … (+, … ; ∘, … ; ∗, …).

… $(X_{ij} - \hat{S}_i(t_{ij}))^2$ … 0, … .001 … $\hat{R}_{ij}^2$ …

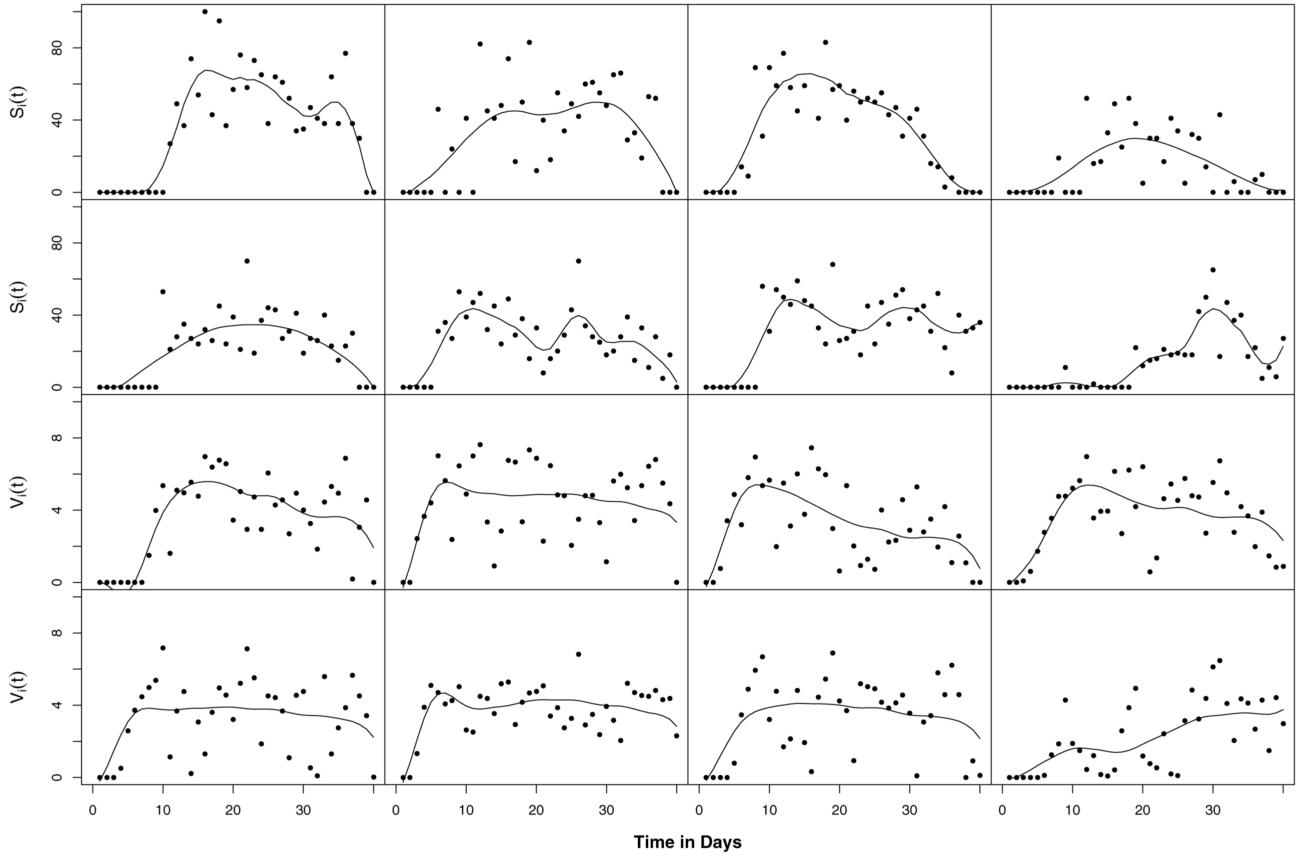
… V_i … 1 … 3. … 1. …

… $\hat{\zeta}_{ik}$, $k = 1, 2$, $i = 1, \ldots, 15$ … (8) … V, … 4( ), … S (… 1991, … 2003) … 4( ). … V_i … S_i. … V_i … S, … 4( ), … S, … $(\hat{\zeta}_{i1}, \hat{\zeta}_{i2})$ … S … 7 … (… 15 …), … 0 … S, …

… $\hat{\zeta}_{ik}$, $k = 1, 2$, … $V_i(t)$. …

5.2 E… -L… D…

…



F 5. (•) F 40 D (R^2) (•) (13)

[illegible]

APPENDIX A: ESTIMATION PROCEDURES

$\kappa_1(\cdot)$ [illegible] $\kappa_2(\cdot,\cdot)$ [illegible] (2.1) [illegible] (2.2). $b_V = b_V(n)$ [illegible] $h_V = h_V(n)$ [illegible] μ_V (4) [illegible] G_V (5) [illegible] 1 [illegible] 2 [illegible] Z_{ij}.

[illegible] ([illegible] 1996) [illegible] $S_i,\ i = 1, \ldots, n,$ [illegible] (t_{ij}, X_{ij}), $j = 1, \ldots, m,$ [illegible] $b_{S,i}$ [illegible]

$$\sum_{j=1}^{m} \kappa_1\left(\frac{t_{ij} - t}{b_{S,i}}\right)\{X_{ij} - \beta_{i,0} - \beta_{i,1}(t - t_{ij})\}^2 \qquad (.1)$$

[illegible] $\beta_{i,0}$ [illegible] $\beta_{i,1}$. [illegible] $\hat{S}_i(t_{ij}) = \hat{\beta}_{i,0}(t_{ij})$. [illegible] $b_{S,i}$ [illegible] b_S [illegible] (2.1). [illegible] μ_V, [illegible]

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \kappa_1\left(\frac{t_{ij} - t}{b_V}\right)\{\hat{Z}_{ij} - \beta_0 - \beta_1(t - t_{ij})\}^2 \qquad (.2)$$

[illegible] β_0 [illegible] β_1, [illegible] $\hat{\mu}_V(t) = \hat{\beta}_0(t)$. [illegible] $_i(t_{ij_1}, t_{ij_2}) = (\hat{Z}_i(t_{ij_1}) - \hat{\mu}_V(t_{ij_1}))(\hat{Z}_i(t_{ij_2}) - \hat{\mu}_V(t_{ij_2}))$, [illegible] $G_V(s,t)$ [illegible]

$$\sum_{i=1}^{n}\sum_{1\le j_1 \ne j_2 \le m} \kappa_2\left(\frac{t_{ij_1} - s}{h_V}, \frac{t_{ij_2} - t}{h_V}\right)$$

$$\times \left\{ {}_i(t_{ij_1}, t_{ij_2}) - f(\beta, (s,t), (t_{ij_1}, t_{ij_2}))\right\}^2, \qquad (.3)$$

[illegible] $f(\beta, (s,t), (t_{ij_1}, t_{ij_2})) = \beta_0 + \beta_{11}(s - t_{ij_1}) + \beta_{12}(t - t_{ij_2})$, [illegible] $\beta = (\beta_0, \beta_{11}, \beta_{12})$, [illegible] $\hat{G}_V(s,t) = \hat{\beta}_0(s,t)$. [illegible] $\{\rho_k, \psi_k\}_{k\ge 1}$ [illegible] $\{\hat{\rho}_k, \hat{\psi}_k\}_{k\ge 1}$ [illegible]

$$\int_{\mathcal{T}} \hat{G}_V(s,t)\hat{\psi}_k(s)\, ds = \hat{\rho}_k \hat{\psi}_k(t), \qquad (.4)$$

[illegible] $\{\hat{\psi}_k\}_{k\ge 1}$ [illegible] ([illegible] 2003). [illegible] M [illegible] ζ_{ik} (8),

$$\hat{\zeta}_{ik} = \sum_{j=2}^{m} (\hat{Z}_{ij} - \hat{\mu}_V(t_{ij}))\hat{\psi}_k(t_{ij})(t_{ij} - t_{i,j-1}),$$

$$i = 1, \ldots, n, k = 1, \ldots, M. \qquad (.5)$$

$\hat{\mu}_V^{(-i)}$ [illegible] $\hat{\psi}_k^{(-i)}$ [illegible] i [illegible]

$$_V(M) = \sum_{i=1}^{n}\sum_{j=1}^{m} \{\hat{Z}_{ij} - \hat{V}_i^{(-i)}(t_{ij})\}^2, \qquad (.6)$$

[illegible] M, [illegible] $\hat{V}_i^{(-i)}(t) = \hat{\mu}_V^{(-i)}(t) + \sum_{k=1}^{M} \hat{\zeta}_{ik}^{(-i)}\hat{\psi}_k^{(-i)}(t)$ [illegible] $\hat{\zeta}_{ik}^{(-i)}$ [illegible] (.5). [illegible] V_i [illegible] (13).

[illegible] σ_W^2, [illegible] G_V, [illegible] $\hat{G}_V^{*}(t)$ [illegible] $\{G_V(t,t) + \sigma_W^2\}$ [illegible] $\hat{Q}_V(s)$, [illegible] b_{Q_V}. [illegible] $\mathcal{T} = [a_1, a_2]$, [illegible] $|\mathcal{T}| = a_2 - a_1$ [illegible] $\mathcal{T}_1 = [a_1 + |\mathcal{T}|/4, a_2 - |\mathcal{T}|/4]$. [illegible]

$$\hat{\sigma}_W^2 = \frac{1}{|\mathcal{T}_1|}\int_{\mathcal{T}_1} \{\hat{Q}_V(t) - \hat{G}_V^{*}(t)\}_+ \, dt \qquad (.7)$$

[illegible] $\hat{\sigma}_W^2 > 0$ [illegible] $\hat{\sigma}_W^2 = 0$ [illegible] $|\mathcal{T}|/4$ [illegible] ([illegible] 2003).

APPENDIX B: ASSUMPTIONS AND NOTATIONS

[illegible] S [illegible] V [illegible]:

(1) [illegible] $C > 0$ [illegible] S [illegible] V [illegible]

$$\sup_t \left|S^{(\nu)}(t)\right| < C \quad \nu = 0, 1, 2$$

$$\sup_t |V(t)| < C.$$

[illegible] $b_{S,i} = b_{S,i}(n)$, $b_V = b_V(n)$, $h_V = h_V(n)$, [illegible] $b_{Q_V} = b_{Q_V}(n)$ [illegible] $\hat{S}_i$ [illegible] (.1) , $\hat{\mu}_V$ [illegible] (.2) , $\hat{G}_V$ [illegible] (.3) , [illegible] $\hat{Q}_V(t)$ [illegible] (.7) . [illegible] n [illegible] m [illegible]:

(2.1) [illegible] $b_{S,i}$, [illegible] b_S [illegible] c_1 [illegible] c_2, $0 < c_1 < \inf_i b_{S,i}/b_S \le \sup_i b_{S,i}/b_S < c_2 < \infty$.

(2.2) $m \to \infty$, $b_S \to 0$, [illegible] $mb_S^2 \to \infty$.

(2.3) $b_V \to 0$, $b_{Q_V} \to 0$, $nb_V^4 \to \infty$, $nb_{Q_V}^4 \to \infty$, [illegible] $n \times b_V^6 < \infty$, [illegible] $nb_{Q_V}^6 < \infty$.

(2.4) $h_V \to 0$, $nh_V^6 \to \infty$, [illegible] $nh_V^8 < \infty$.

(2.5) [illegible] $n^{1/2}b_V m^{-1} < \infty$, [illegible] $n^{1/2}b_{Q_V} m^{-1} < \infty$, [illegible] $n^{1/2}h_V m^{-1} < \infty$.

[illegible] $\{t_{ij}\}_{i=1,\ldots,n; j=1,\ldots,m}$ [illegible] i [illegible] $j = 1, \ldots, m-1$, $t_{ij} < t_{i,j+1}$ [illegible] f [illegible] $\int_{\mathcal{T}} f(t)\, dt = 1$ [illegible] $f(t) > 0$ [illegible] t_{ij} [illegible] $t_{ij} = F^{-1}(\frac{j-1}{m-1})$, [illegible] F^{-1} [illegible] $F(t) = \int_{a_1}^{t} f(s)\, ds$. [illegible] t_{ij} [illegible] i, [illegible] c_1 [illegible] c_2 [illegible] f_i [illegible] $0 < c_1 < \inf_i \inf_{t\in\mathcal{T}} f_i(t) < \sup_i \sup_{t\in\mathcal{T}} f_i(t) < c_2$. [illegible] N_i [illegible] i [illegible] $m \to \infty$ [illegible] c_1 [illegible] c_2, $0 < c_1 < \inf_i \frac{N_i}{m} < \sup_i \frac{N_i}{m} < c_2 < \infty$; [illegible] $N_i = m$, [illegible] m [illegible].

$n = \{t_{ij} - t_{i,j-1} : j = 2, \ldots, m\}$,

(3) $n = O(m^{-1})$, $n, m \to \infty$.

X_{ij} Z_{ij} $t \in \mathcal{T}$,

(4) $E[X_{ij}^4] < \infty$ $E[Z_{ij}^4] < \infty$.

H H (1989). $(f \otimes g)(h) = \langle f, h \rangle g$ $f, g, h \in H$, H H H $F \equiv \sigma_2(H)$, $\langle T_1, T_2 \rangle_F = (T_1 T_2^*) = \sum_j \langle T_1 u_j, T_2 u_j \rangle_H$ $\|T\|_F^2 = \langle T, T \rangle_F$, $T_1, T_2, T \in F$, T_2^* T_2, $\{u_j : j \geq 1\}$ H. $\mathbf{G}_V$ (6) $\hat{\mathbf{G}}_V$, G_V (5), $\hat{G}_V$ (.3), H $\mathcal{I}_i = \{j : \rho_j = \rho_i\}$ $\mathcal{I}' = \{i : |\mathcal{I}_i| = 1\}$, $|\mathcal{I}_i|$ $\mathcal{I}_i$. $\mathbf{P}_j^V = \sum_{k \in \mathcal{I}_j} \psi_k \otimes \psi_k$ $\hat{\mathbf{P}}_j^V = \sum_{k \in \mathcal{I}_j} \hat{\psi}_k \otimes \hat{\psi}_k$ H $\{\psi_k : k \in \mathcal{I}_j\}$ $\{\hat{\psi}_k : k \in \mathcal{I}_j\}$. j,

$$\delta_j^V = \frac{1}{2} \min\{|\rho_l - \rho_j| : l \notin \mathcal{I}_j\}, \qquad (.1)$$

$\mathbf{\Lambda}_{\delta_j^V} = \{z \in \mathcal{C} : |z - \rho_j| = \delta_j^V\}$, $\mathcal{C}$ $\mathbf{G}_V$ $\hat{\mathbf{G}}_V$ $\mathbf{R}_V$ $\hat{\mathbf{R}}_V$, $\mathbf{R}_V(z) = (\mathbf{G}_V - zI)^{-1}$ $\hat{\mathbf{R}}_V(z) = (\hat{\mathbf{G}}_V - zI)^{-1}$.

$$A_{\delta_j^V} = \sup\{\|\mathbf{R}_V(z)\|_F : z \in \mathbf{\Lambda}_{\delta_j^V}\} \qquad (.2)$$

$M = M(n)$ $V(t)$, $\hat{V}_i(t) = \hat{\mu}_V(t) + \sum_{m=1}^{M(n)} \hat{\zeta}_{im} \hat{\psi}_m(t)$ (13). $\|\pi\|_\infty = \sup_{t \in \mathcal{T}} |\pi(t)|$ $\pi(\cdot)$ $\mathcal{T}$. μ_V ψ_j $M = M(n)$ n m $n \to \infty$,

(5) $\tau_n = \sum_{j=1}^{M} (\delta_j^V A_{\delta_j^V} \|\psi_j\|_\infty) / (\sqrt{n} h_V^2 - A_{\delta_j^V}) \to 0$, $M = M(n) \to \infty$;

(6) $\sum_{j=1}^{M} \|\psi_j\|_\infty = o(\min\{\sqrt{n} b_V, \sqrt{m}\})$ $\sum_{j=1}^{M} \|\psi_j\|_\infty \times \|\psi_j'\|_\infty = o(m)$.

(5) (6) ψ_j j $\theta_m = b_S^2 + (\sqrt{m} b_S)^{-1}$, V

(7) $E\{[\sup_{t \in \mathcal{T}} |V(t) - V^{(M)}(t)|]^2\} = o(n)$, $V^{(M)}(t) = \mu_V(t) + \sum_{k=1}^{M} \zeta_k \psi_k(t)$.

(8) $1 \leq i \leq n$, $\theta_m \sum_{k=1}^{M} \|\psi_k\|_\infty^2 = o_p(1)$ $\gamma_n = \sum_{k=1}^{M} (\delta_k^V A_{\delta_k^V} \|\psi_k\|_\infty) / (\theta_m^{-1} - A_{\delta_k^V}) \to 0$ $n \to \infty$.

$t = t_{ij}, t_1 = t_{ij_1}$, $t_2 = t_{ij_2}$, i, j, j_1, j_2, $g(x; t)$ X_{ij} $g_2(x_1, x_2; t_1, t_2)$ (X_{ij_1}, X_{ij_2}). $f(z; t)$ $f_2(z_1, z_2; t_1, t_2)$ Z_{ij} (Z_{ij_1}, Z_{ij_2}). $g(\cdot; t), f(\cdot; t), t \in \mathcal{T}$, $g_2(\cdot; t_1, t_2), f_2(\cdot; t_1, t_2), t_1, t_2 \in \mathcal{T}$,

(1.1) $(d^2/dt^2) g(x; t)$ $(d^2/dt^2) f(z; t)$ $\times \mathcal{T}$.

(1.2) $(d^2/dt_1{}^1 dt_2{}^2) g_2(x_1, x_2; t_1, t_2)$ $(d^2/dt_1{}^1 dt_2{}^2) f_2(z_1, z_2; t_1, t_2)$ $^2 \times \mathcal{T}^2$, $_1 + {}_2 = 2, 0 \leq {}_1, {}_2 \leq 2$.

$\kappa_1 : \to$ $\kappa_2 : {}^2 \to$ $\kappa_1(u)$ $\kappa_2(u, v)$ $\chi_1(t) = \int e^{-iut} \kappa_1(u)\, du$ $\chi_2(t, s) = \int e^{-(iut + ivs)} \kappa_2(u, v)\, du\, dv$.

(2.1) κ_1 $\|\kappa_1\|^2 = \int \kappa_1^2(u)\, du < \infty$, $\chi_1(t)$ $\int |\chi_1(t)|\, dt < \infty$.

(2.2) κ_2 $\|\kappa_2\|^2 = \int \int \kappa_2^2(u, v)\, du\, dv < \infty$, κ_2 0 u v. $\chi_2(t, s)$ $\int \int |\chi_2(t, s)|\, dt\, ds < \infty$.

APPENDI C: PROOFS

P 1

W S V, $= {}_1 \times {}_2$. E^* ^{2} S V), $\omega_1 \in {}_1$. ω_1, $R_j = R_j(\omega_1)$ i R_{ij} (1) ^{2} j, $E^*(R_j) = 0$ $E^*(R_j^2) < C_1$ C_1. (1) ^{3} ^{2} (1979)

$$E^*\Big(\sup_{t \in \mathcal{T}} |\hat{S}(t) - S(t)|\Big)(\omega_1) = O\Big(b_S^2 + \frac{1}{\sqrt{m} b_S}\Big)(\omega_1).$$

ω_1, $|S^{(v)}(\omega_1)|, v = 0, 1, 2$, $|V(\omega_1)|$ (1), ω_1;

$$\sup_{\omega_1 \in {}_1} E^*\Big(\sup_{t \in \mathcal{T}} |\hat{S}(t) - S(t)|\Big)(\omega_1) = O\Big(b_S^2 + \frac{1}{\sqrt{m} b_S}\Big),$$

(14).

.1. V $\{t_{ij}, Z_{ij}\}$ ( ), $\tilde{\mu}_V, \tilde{G}_V, \tilde{\sigma}_W^2, \tilde{\psi}_k, \tilde{\rho}_k$, $\tilde{\zeta}_{ik}$, (.2), (.3), (.4), (.5)

$$\alpha_{nk} = \frac{\delta_k^V A_{\delta_k^V}}{\sqrt{n} h_V^2 - A_{\delta_k^V}} \qquad \alpha_{nk}^* = \frac{\delta_k^V A_{\delta_k^V}}{\theta_m^{-1} - A_{\delta_k^V}}, \qquad (.1)$$

$\theta_m = b_S^2 + (\sqrt{m} b_S)^{-1}$ δ_k^V $A_{\delta_k^V}$ (.1) (.2).

Lemma C.1. (2.1), (2.3), (3), (5), (1.1), (2.2),

$$\sup_{t \in \mathcal{T}} |\tilde{\mu}_V(t) - \mu_V(t)| = O_p\Big(\frac{1}{\sqrt{n} b_V}\Big)$$

$$\sup_{s,t \in \mathcal{T}} |\tilde{G}_V(s, t) - G_V(s, t)| = O_p\Big(\frac{1}{\sqrt{n} h_V^2}\Big). \qquad (.2)$$

ρ_k 1, $\hat{\psi}_k$

$$\sup_{t \in \mathcal{T}} |\tilde{\psi}_k(t) - \psi_k(t)| = O_p(\alpha_{nk}) \qquad (\ .3)$$

$$|\tilde{\rho}_k - \rho_k| = O_p(\alpha_{nk}),$$

$\alpha_{nk} \to 0$ $n \to \infty$, k, α_{nk} (.1), $O_p(\cdot)$ (.3) $1 \le k \le M$. (.2),

$$\sup_{t \in \mathcal{T}} |\tilde{\sigma}_W^2(t) - \sigma_W^2(t)| = O_p\left(\left\{ \frac{1}{\sqrt{n} h_V^2}, \frac{1}{\sqrt{n} b_{Q_V}} \right\} \right). \qquad (\ .4)$$

(1), (7) (1.1), (2.2),

$$\sup_{1 \le k \le M} |\tilde{\zeta}_{ik} - \zeta_{ik}| \xrightarrow{p} 0 \qquad (\ .5)$$

$$\sup_{t \in \mathcal{T}} \left| \sum_{k=1}^{M} \tilde{\zeta}_{ik} \tilde{\psi}_k(t) - \sum_{k=1}^{\infty} \zeta_{ik} \psi_k(t) \right| \xrightarrow{p} 0,$$

M $M = M(n) \to \infty$ $n \to \infty$.

Proof of Lemma C.1. (.2), (.3), (.5) 1 1 (2006). $\tilde{\sigma}^2_{W\sigma_t}$

$$+ \sup_{t \in \mathcal{T}} \left\{ \left| \sum_{k=1}^{M} \tilde{\zeta}_{ik} \tilde{\psi}_k(t) - \sum_{k=1}^{\infty} \zeta_{ik} \psi_k(t) \right| \right\}$$

$$\equiv Q_{i1}(n) + Q_{i2}(n),$$

$Q_{i1}(n) \xrightarrow{p} 0$ $Q_{i2}(n) \xrightarrow{p} 0.$ (.5), $Q_{i2}(n) \xrightarrow{p} 0$ (1), (7), $Q_{i2}(n) = O(\vartheta_{in}^{(2)})$, $O(\cdot)$ $1 \le i \le n$. $Q_{i1}(n)$,

$$Q_{i1}(n) \le \sup_{t \in \mathcal{T}} \left\{ \sum_{k=1}^{M} |\hat{\zeta}_{ik} - \tilde{\zeta}_{ik}| \cdot |\hat{\psi}_k(t)| + \sum_{k=1}^{M} |\tilde{\zeta}_{ik}| \cdot |\hat{\psi}_k(t) - \tilde{\psi}_k(t)| \right\}. \quad (.11)$$

(.10), (.11)

$$C_1 \theta_{im} \sum_{k=1}^{M} \|\psi_k\|_\infty^2 + \tilde{\gamma}_n \left\{ C_2 + \sum_{j=2}^{m} |Z_{ij}| (t_{ij} - t_{i,j-1}) \right\} \xrightarrow{p} 0.$$

(.11) $O_p\{\sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} E|\zeta_{ik}|/(\theta_m^{-1} - A_{\delta_k^V})\}$. $E\{\sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} E|\zeta_{ik}|/(\theta_m^{-1} - A_{\delta_k^V})\} \le \sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} \sqrt{\rho_k}/(\theta_m^{-1} - A_{\delta_k^V}) \le \gamma_n$, $\rho_k \to 0$, 0 $Q_{i1}(n) = O_p(\vartheta_{in}^{(1)})$, $O_p(\cdot)$ i. (18) $\sup_{t \in \mathcal{T}} |\hat{V}_i(t) - V_i(t)| = O_p(\vartheta_{in}^{(1)} + \vartheta_{in}^{(2)})$, $O_p(\cdot)$ $1 \le i \le n$.

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