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1. INTRODUCTION

Nonparametric regression is a well-known technique for estimating a function g based on a sample (t_j, Y_j) , $j = 1, \dots, J$. A classical approach to nonparametric regression is to assume that the observations (t_j, Y_j) are independent and identically distributed (i.i.d.) random variables. This approach has been studied extensively in the literature (cf., e.g., Härdle et al., 1991; Müller and Wang, 1996; Müller, 1997; Müller and Wang, 1998; Müller, 1999).

Another approach to nonparametric regression is to assume that the observations (t_j, Y_j) are dependent. This approach has been studied by several authors (cf., e.g., Müller, 1974; Müller and Stoyan, 1995; Müller and Stoyan, 1996; Müller and Stoyan, 1986; Müller and Stoyan, 1987; Müller and Stoyan, 1993; Müller and Stoyan, 1998; Müller and Stoyan, 1998; Müller and Stoyan, 2000; Müller and Stoyan, 2004).

$$Y_j = g(t_j) + e_j(t_j), \quad j = 1, \dots, J.$$

$$(t_j)_{j=1,\dots,J} \text{ is a realization of } X, \quad v(t_j) = \text{var}(e_j(t_j))$$

where (X, Y) is a bivariate random variable and $v(x) = E(Y^2 | X=x) - E(Y | X=x)^2$.

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It is well known that the function g can be estimated by the Nadaraya-Watson estimator (Nadaraya, 1964; Watson, 1964) if the observations (t_j, Y_j) are i.i.d. random variables. The Nadaraya-Watson estimator is given by

$$\hat{g}_N(t) = \frac{\sum_{j=1}^J Y_j K\left(\frac{t-t_j}{h}\right)}{\sum_{j=1}^J K\left(\frac{t-t_j}{h}\right)}, \quad (1)$$

where K is a kernel function and h is a bandwidth. The Nadaraya-Watson estimator is a local linear estimator (cf., e.g., Müller and Wang, 1996; Müller, 1997; Müller and Wang, 1998).

It is well known that the Nadaraya-Watson estimator is not consistent if the observations (t_j, Y_j) are dependent. This is because the Nadaraya-Watson estimator is a local linear estimator and it does not take into account the dependence structure of the observations. In fact, the Nadaraya-Watson estimator is a local linear estimator and it does not take into account the dependence structure of the observations.

In this paper, we propose a new nonparametric regression estimator that takes into account the dependence structure of the observations. The proposed estimator is a local linear estimator and it is consistent under general conditions.

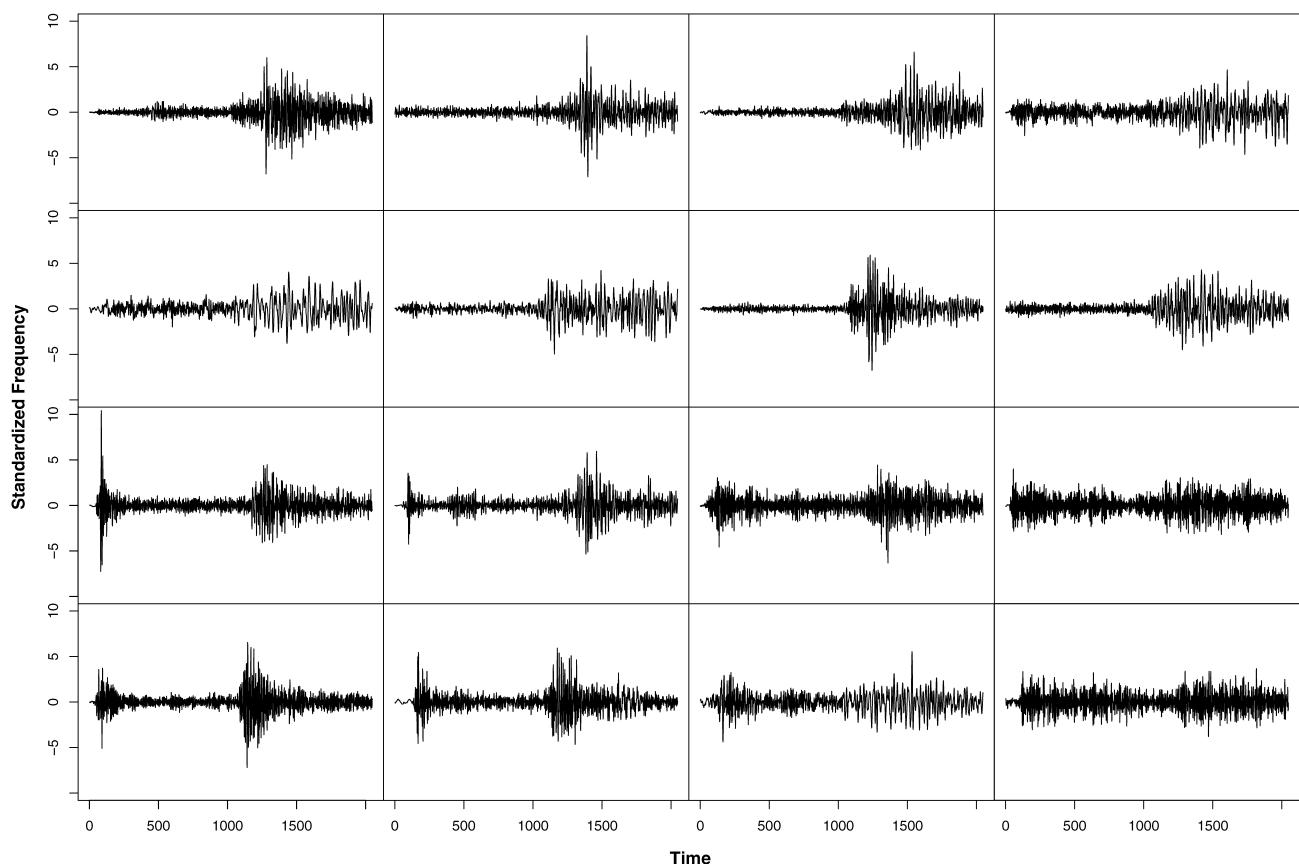
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1986). β^T (1950), (1958), (1991; 1992; 2005; 2001; 2003).

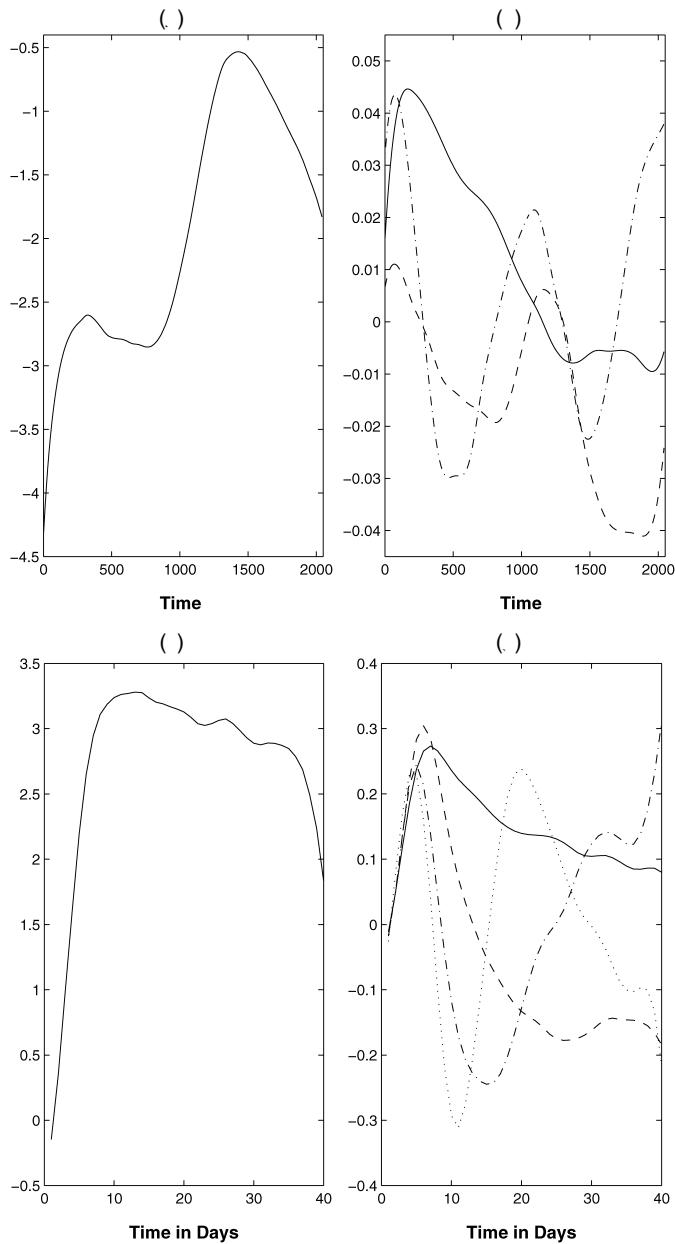
2

3, 4, 1), 2, 3), 5,

2. DECOMPOSING FUNCTIONAL DATA

S_i, \dots, S_n V_i, \dots, V_n U_i, \dots, U_n $S_i, i = 1, \dots, n$

X_{ij}

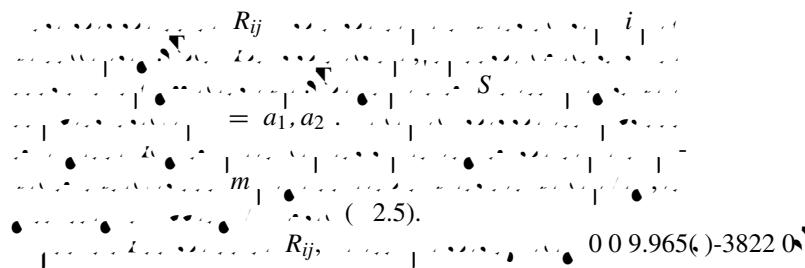


$F_1 = 2. C_n \dots F_2 = E_n \dots P = E_1, L_1, D_1$
 $(\dots), (\dots), (\dots), (\dots), (\dots), (\dots), (\dots)$
 \dots
 $n = 0.025$

$$t_{ij} = t_{ij}, \quad i = 1, \dots, n, j = 1, \dots, m.$$

$$\sum_{i \neq i'} R_{ij} = 1, \quad R_{ij} > 0, \quad R_{ij} \in S_i, \quad i = 1, \dots, n, j = 1, \dots, m.$$

$$ER_{ij} = 0, \quad (R_{ij}) = \frac{2}{R_{ij}} < \infty.$$



$$\begin{aligned} (Z_{ij}, Z_{ik}) &= (V_i(t_{ij}), V_i(t_{ik})) \\ &= G_V(t_{ij}, t_{ik}), \quad j \neq k. \end{aligned} \quad (9)$$

$$G_V(t_{ij}, t_{ik}) = \frac{1}{W} \int_{t_{ij}}^{t_{ik}} V_i(s) ds, \quad (1998)$$

$$\mu_V = G_V(0, T), \quad (1991)$$

$S_k(t) = \sum_{k=1}^{\infty} \mu_k S_k(t)$, $k=1, 2, \dots, n$, $\sum_k \mu_k < \infty$, $E(S_k) = 0$, $(S_k) = \mu_k$, $E(S_k^2) = \mu_k^2$

$$S(t) = \mu_S(t) + \sum_{k=1}^{\infty} S_k(t) \quad (10)$$

$$V(t) = \mu_V(t) + \sum_{k=1}^{\infty} V_k(t).$$

$W_{ij} = X_{ij} - S_i(t_{ij}) - V_i(t_{ij})$, $i = 1, \dots, n$, $j = 1, \dots, m$, $E(W_{ij}) = 0$, $(W_{ij}) = \frac{1}{W}$, $P(W_{ij} > 0) = P(W_{ij} < 0) = \frac{1}{2}$.

$$X_{ij} = S_i(t_{ij}) + V_i(t_{ij}) \{ V_i(t_{ij}) + W_{ij} \}^{1/2}. \quad (11)$$

$S_i(t_{ij}) = \sum_{k=1}^{\infty} S_{ik}(t_{ij})$, $V_i(t_{ij}) = \sum_{k=1}^{\infty} V_{ik}(t_{ij})$, $S_{ik}(t_{ij}) = 0$, $V_{ik}(t_{ij}) = 0$, $k=1, \dots, n$.

3. ESTIMATION OF MODEL COMPONENTS

$Z_{ij} = X_{ij} - S_i(t_{ij}) - V_i(t_{ij})$, $i = 1, \dots, n$, $j = 1, \dots, m$, $(Z_{ij}) = 0$, $(Z_{ij}) = \frac{1}{W}$, $(W_{ij}) = \frac{1}{W}$, $X_{ij} = S_i(t_{ij}) + V_i(t_{ij}) \{ V_i(t_{ij}) + W_{ij} \}^{1/2}$, $i = 1, \dots, n$, $j = 1, \dots, m$, $i \neq j$, $(1), (7)$, (8) , (2003) , (2005) (1998).

1. $Z_{ij} = X_{ij} - S_i(t_{ij}) - V_i(t_{ij})$, $i = 1, \dots, n$, $j = 1, \dots, m$, $\mu_V = G_V(0, T)$, $b_V = \frac{1}{W} \int_0^T V_i(s) ds$, $b_V = \frac{1}{W} \int_0^T V(s) ds$, $R_{ij} = X_{ij} - S_i(t_{ij})$, $Z_{ij} = R_{ij}^2 = (X_{ij} - S_i(t_{ij}))^2$, $i = 1, \dots, n$, $j = 1, \dots, m$. (12)
2. $Z_{ij} = X_{ij} - S_i(t_{ij}) - V_i(t_{ij})$, $i = 1, \dots, n$, $j = 1, \dots, m$, $\mu_V = G_V(0, T)$, $b_V = \frac{1}{W} \int_0^T V_i(s) ds$, $b_V = \frac{1}{W} \int_0^T V(s) ds$, $R_{ij} = X_{ij} - S_i(t_{ij})$, $t_{ij} \neq t_{ij'}$, $Z_{ij} = R_{ij}^2 = (X_{ij} - S_i(t_{ij}))^2$, $i = 1, \dots, n$, $j = 1, \dots, m$. (12)
3. $Z_{ij} = X_{ij} - S_i(t_{ij}) - V_i(t_{ij})$, $i = 1, \dots, n$, $j = 1, \dots, m$, $\mu_V = G_V(0, T)$, $b_V = \frac{1}{W} \int_0^T V_i(s) ds$, $b_V = \frac{1}{W} \int_0^T V(s) ds$, $R_{ij} = X_{ij} - S_i(t_{ij})$, $t_{ij} \neq t_{ij'}$, $Z_{ij} = R_{ij}^2 = (X_{ij} - S_i(t_{ij}))^2$, $i = 1, \dots, n$, $j = 1, \dots, m$. (12)
4. $Z_{ij} = X_{ij} - S_i(t_{ij}) - V_i(t_{ij})$, $i = 1, \dots, n$, $j = 1, \dots, m$, $\mu_V = G_V(0, T)$, $b_V = \frac{1}{W} \int_0^T V_i(s) ds$, $b_V = \frac{1}{W} \int_0^T V(s) ds$, $R_{ij} = X_{ij} - S_i(t_{ij})$, $t_{ij} \neq t_{ij'}$, $Z_{ij} = R_{ij}^2 = (X_{ij} - S_i(t_{ij}))^2$, $i = 1, \dots, n$, $j = 1, \dots, m$. (12)
5. $Z_{ij} = X_{ij} - S_i(t_{ij}) - V_i(t_{ij})$, $i = 1, \dots, n$, $j = 1, \dots, m$, $\mu_V = G_V(0, T)$, $b_V = \frac{1}{W} \int_0^T V_i(s) ds$, $b_V = \frac{1}{W} \int_0^T V(s) ds$, $R_{ij} = X_{ij} - S_i(t_{ij})$, $t_{ij} \neq t_{ij'}$, $Z_{ij} = R_{ij}^2 = (X_{ij} - S_i(t_{ij}))^2$, $i = 1, \dots, n$, $j = 1, \dots, m$. (12)

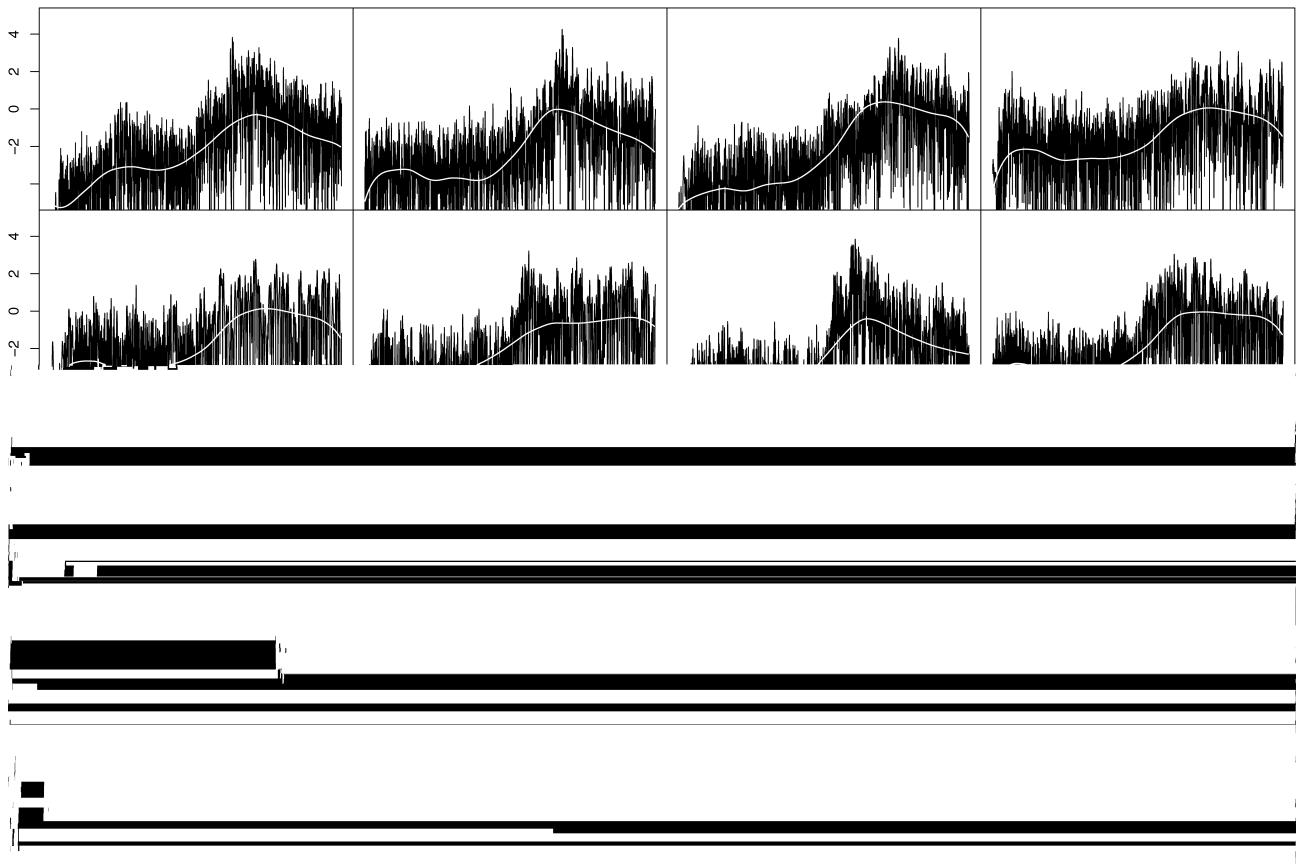


Fig. 3. Observed data (F_t) and estimated data (\hat{F}_t) for E_{n+1} ($n = 1, \dots, 25$) in (13) ($\mu_V = 0.25$, $b_S = 0.1$).

(6). Figure 3 shows the observed data (F_t) and estimated data (\hat{F}_t) for E_{n+1} ($n = 1, \dots, 25$), where $\mu_V = 0.25$, $b_S = 0.1$. The data are plotted in four subplots for $n = 1, 5, 10, 25$. The estimated data (\hat{F}_t) is shown as a smooth curve fitted to the observed data (F_t). The bottom row shows five horizontal bars, each divided into segments of different lengths, representing step functions or piecewise constant signals.

$$V_i(t) = \mu_V(t) + \sum_{k=1}^M b_{ik} s_k(t). \quad (13)$$

Step function approximation of $V_i(t)$ is given by

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$$Z_{ij} = \sum_{k=1}^M b_{ik} s_k(t) \quad (13)$$

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$$S_i = \sum_{k=1}^M b_{ik} s_k(t) \quad (11)$$

$$b_S = \sum_{i=1}^m b_{Si} \quad (10)$$

$s_k(t) = \begin{cases} 1 & \text{if } t \in [t_{k-1}, t_k], \\ 0 & \text{otherwise.} \end{cases}$ (2.1), $b_{Si} = \sum_{k=1}^M b_{ik} s_k(t) = \sum_{k=1}^M b_{ik} \sum_{j=1}^m b_{Sj} \delta_{kj} = b_{Sj} \sum_{k=1}^M b_{ik} = b_{Sj} b_S, \quad (2.2);$

$$b_V = \sum_{i=1}^m b_{Si} = \sum_{i=1}^m b_S s_i(t) = b_S \sum_{i=1}^m s_i(t) = b_S h_V, \quad (2.3),$$

$$h_V = \sum_{i=1}^m s_i(t) = \sum_{i=1}^m \sum_{k=1}^M b_{ik} s_k(t) = \sum_{k=1}^M b_{ik} \sum_{i=1}^m s_i(t) = b_{ik} h_V, \quad (2.4),$$

$$G_V(s, t) = \int_0^t h_V(s) ds = \int_0^t \sum_{i=1}^m s_i(t) ds = \sum_{i=1}^m \int_0^t s_i(t) ds = \sum_{i=1}^m \frac{1}{2} t^2 s_i(t) = \frac{1}{2} t^2 G_V(t), \quad (2.5).$$

Step function approximation of $V_i(t)$ is given by

$$Z_{ij} = \sum_{k=1}^M b_{ik} s_k(t) \quad (13)$$

Theorem 1. If $s_i(t) = 1$ for $t \in [t_{k-1}, t_k]$ and $s_i(t) = 0$ for $t \in [t_k, t_{k+1}]$, then

$$E\left(\sum_{i \in T} S_i(t) - \hat{S}_i(t)\right) = O\left(b_S^2 + \frac{1}{mb_S}\right). \quad (14)$$

$$\begin{aligned} & \mu_V(t) = G_V(s, t), \quad k(t) = k(t), \\ & G_V(s, t) = \frac{1}{\sqrt{\bar{W}}} \sum_{i=1}^n V_i(t), \quad k(t) = \frac{1}{\sqrt{\bar{W}}} \sum_{i=1}^n V_i(t)^2, \\ & (\text{.2}), (\text{.3}), (\text{.4}) \quad \text{and} \quad (\text{.7}) \\ & \frac{2}{\bar{W}} (\text{.2}) \quad \text{and} \quad \frac{2}{\bar{W}} (\text{.7}). \end{aligned}$$

Theorem 2. (1), (8), (1.1), (2.2),

$$\begin{aligned} & \mu_V(t) - \mu_V(t) \\ & = O_p \left(b_S^2 + \frac{1}{\bar{m}b_S} + \frac{1}{\bar{n}b_V} \right), \\ & G_V(s, t) - G_V(s, t) \\ & = O_p \left(b_S^2 + \frac{1}{\bar{m}b_S} + \frac{1}{\bar{n}h_V^2} \right), \quad (15) \\ & = O_p \left(b_S^2 + \frac{1}{\bar{m}b_S} + \frac{1}{\bar{n}h_V^2} + \frac{1}{\bar{n}b_{QV}} \right). \end{aligned}$$

$$\begin{aligned} & k(t) - k(t) \xrightarrow{p} 0, \quad k \xrightarrow{p} k. \quad (16) \end{aligned}$$

$$\begin{aligned} & k(t) = O_p \left(\frac{1}{nk} + \frac{1}{nk} \right), \quad k(t) = O_p \left(\frac{1}{nk} + \frac{1}{nk} \right), \\ & \frac{1}{nk} (\text{.1}) \quad \text{and} \quad \frac{1}{nk} (\text{.2}). \end{aligned}$$

$$\begin{aligned} & V_i \quad (\text{.3}) \quad \text{and} \quad \text{.3}, \\ & V_i, \quad (\text{.5}) \quad \text{and} \quad \text{.5}, \\ & ik \quad (\text{.5}) \quad \text{and} \quad V_i, \\ & ik \quad (\text{.5}) \quad \text{and} \quad V_i. \end{aligned}$$

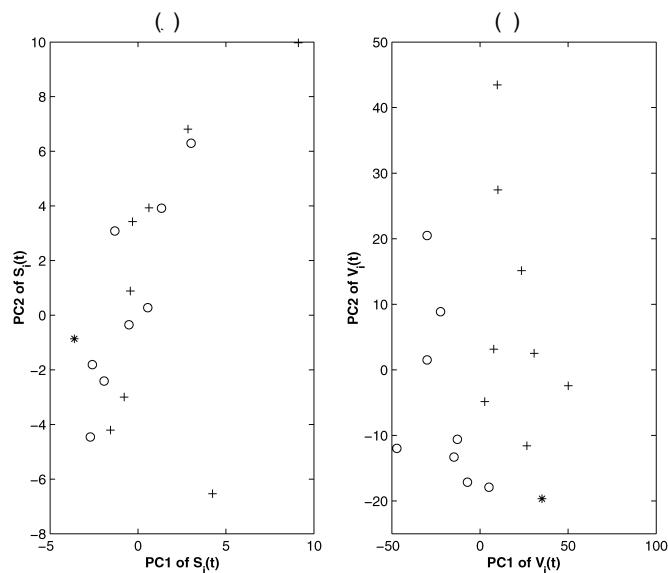
Theorem 3. (1), (8), (1.1), (2.2),

$$\begin{aligned} & V_i \quad (\text{.5}) \quad \text{and} \quad V_i, \\ & ik \quad (\text{.5}) \quad \text{and} \quad ik \xrightarrow{p} 0, \quad (17) \end{aligned}$$

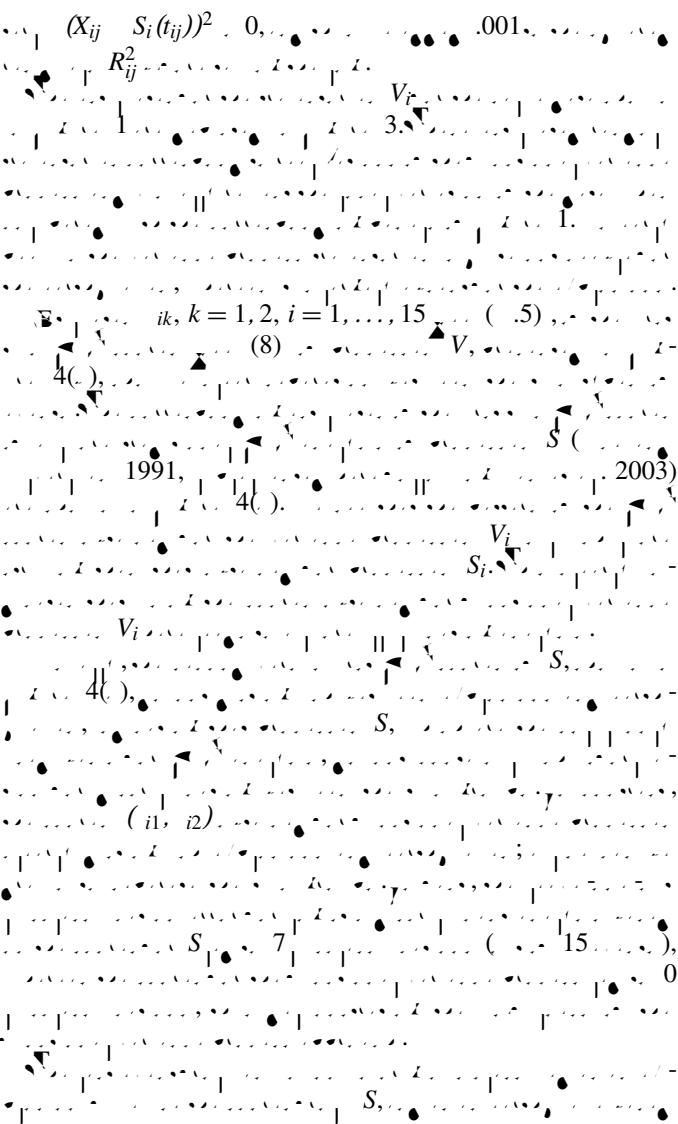
$$\begin{aligned} & (13), M = \\ & M(n) \rightarrow \infty, \quad n \rightarrow \infty, \\ & V_i(t) \quad (\text{.5}) \quad \text{and} \quad V_i, \\ & 1 \leq i \leq n, \end{aligned}$$

$$\begin{aligned} & V_i(t) - V_i(t) \xrightarrow{p} 0. \quad (18) \end{aligned}$$

(16), (17), (18)



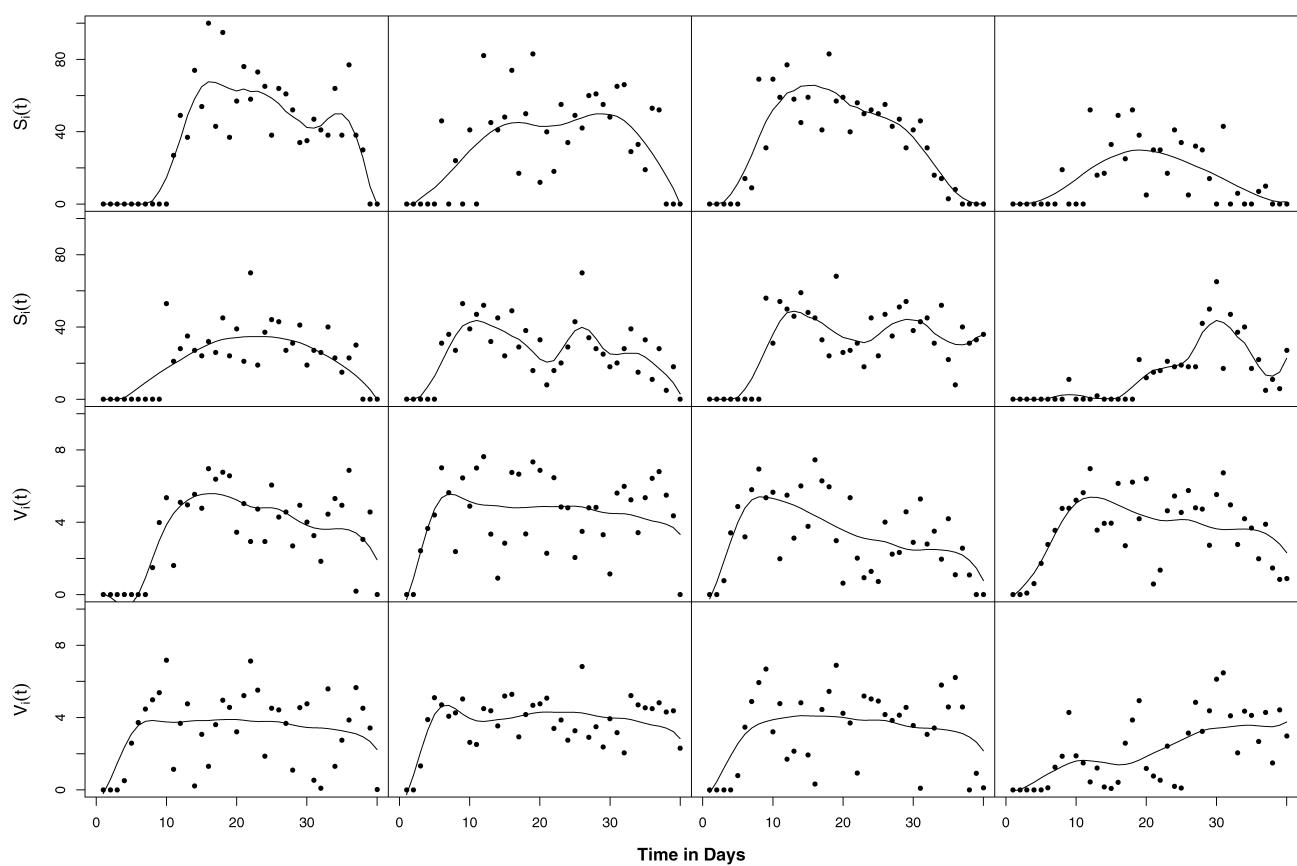
F, *C*, *n*, *PC2*, *E*, *P*, *D*, *S*, *V*, *i*, *k*, *t*, *Q*, *R*, *PC1*, *O*, *Q*, *P*, *E*, *D*, *(+)*, *(*)*, *(○)*, *(■)*.



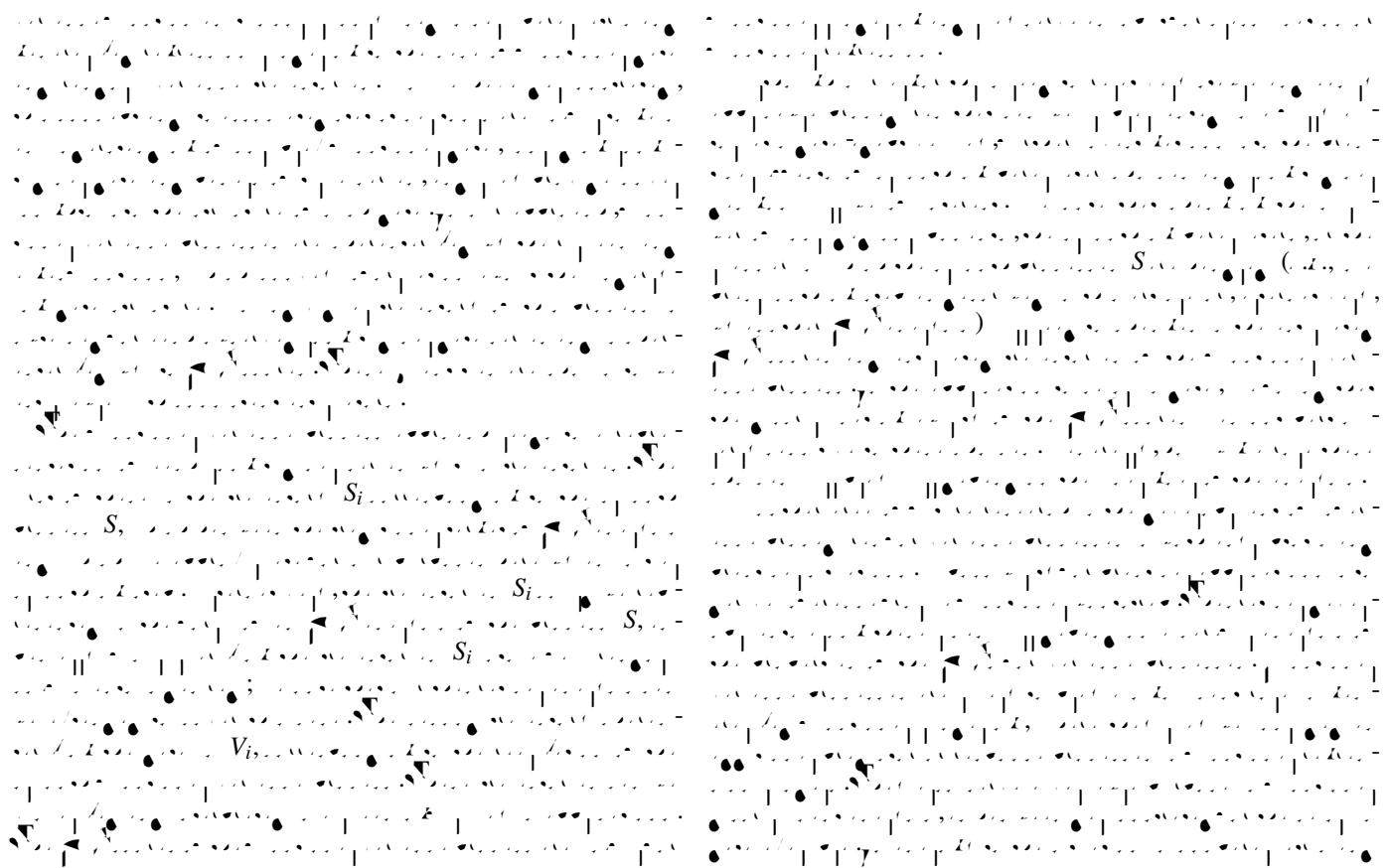
$\sum_{k=1}^{15} \sum_{i=1}^{15} \sum_{j=1}^{15} \sum_{l=1}^{15} \sum_{m=1}^{15} \sum_{n=1}^{15} \sum_{p=1}^{15} \sum_{q=1}^{15} \sum_{r=1}^{15} \sum_{s=1}^{15} \sum_{t=1}^{15} \sum_{u=1}^{15} \sum_{v=1}^{15} \sum_{w=1}^{15} \sum_{x=1}^{15} \sum_{y=1}^{15} \sum_{z=1}^{15} \sum_{ik} V_i(t_k) V_j(t_l) V_m(t_n) V_p(t_s) V_q(t_t) V_r(t_u) V_w(t_x) V_z(t_y) = 0$

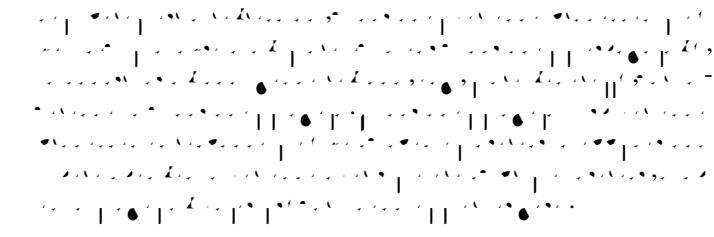
5.2 E₁₁-L₁₁-D₁₁

$$\begin{aligned} & E_{11} = 50.8(+) - 410.7(-) - 0.2(+) + 8(+) - 5(-) + 52(-) \\ & L_{11} = 100(+) - 100(-) + 100(+) - 100(-) + 100(+) - 100(-) \\ & D_{11} = 100(+) - 100(-) + 100(+) - 100(-) + 100(+) - 100(-) \end{aligned}$$



$F_1 = 5.0 \cdot E_1 - L_1, C_1 = (\dots) \cdot S_1 - L_1, \dots, S_n = R_n - n \cdot F_n + P_n, E_n = E_1 \cdot M_n + \dots + E_n \cdot M_n = (R^2(\dots)) \cdot (S_n - \dots - S_n).$





APPENDI A: ESTIMATION PROCEDURE

$$(1) \quad b_V = b_V(n), \quad h_V = h_V(n), \quad \mu_V = \mu_V(n) \quad (2.1)$$

$$(2) \quad G_V = G_V(t), \quad 1, 2, \dots, n, \quad (2.2)$$

$$Z_{ij} = \sum_{j=1}^m$$

(1996).
and $S_i, i = 1, \dots, n$, $t_{ij}, X_{ij},$
 $j = 1, \dots, m$, $b_{S,i}$

$$\sum_{j=1}^m 1 \left(\frac{t_{ij} - t}{b_{S,i}} \right) \{X_{ij} - s_{i,0} - s_{i,1}(t - t_{ij})\}^2 \quad (1)$$

$s_{i,0}(t_{ij}) = \frac{s_{i,0} - s_{i,1}(t_{ij})}{b_{S,i}}$ and $b_{S,i} = b_{S,i}(n)$ (2.1).

$$\mu_V, \quad (2.2) \quad Z_{ij} = \sum_{i=1}^n \sum_{j=1}^m$$

$$\sum_{i=1}^n \sum_{j=1}^m 2 \left(\frac{t_{ij} - s}{h_V} \right) \{Z_{ij} - s_{i,0} - s_{i,1}(t - t_{ij})\}^2 \quad (2)$$

$$\mu_V(t) = \mu_V(t_{ij_1}) + \mu_V(t_{ij_2}) = Z_{ij_1} - \mu_V(t_{ij_1})(Z_{ij_2} - \mu_V(t_{ij_2})) \quad (2.3)$$

$$\sum_{k=1}^n f_{ik}(s, t, (t_{ij_1}, t_{ij_2})) = f_{00}(s - t_{ij_1}) + f_{12}(t - t_{ij_2}),$$

$$f_{00} = (f_{00}, f_{11}, f_{12}), \quad f_{12} = (f_{11}, f_{12}), \quad G_V(s, t) = g_0(s, t).$$

$$\sum_{k=1}^n f_{ik}(s, t, (t_{ij_1}, t_{ij_2})) = \sum_{k=1}^n f_{ik}(s, t, (t_{ij_1}, t_{ij_2})) \quad (2.4)$$

$$\int_T G_V(s, t) f_{ik}(s) ds = f_{ik}(t), \quad (2.5)$$

$$\{f_{ik}\}_{i=1, \dots, n, k=1, \dots, m} \quad (2003).$$

$$M \quad \sum_{i=1}^n \sum_{j=1}^m f_{ik}(t_{ij}) \quad (2.6)$$

$$f_{ik} = \sum_{j=2}^m (Z_{ij} - \mu_V(t_{ij})) f_{ik}(t_{ij}) (t_{ij} - t_{ij-1}),$$

$$i = 1, \dots, n, k = 1, \dots, M. \quad (2.7)$$

$$\mu_V^{(i)} = \sum_{k=1}^M f_{ik},$$

$$V_M = \sum_{i=1}^n \sum_{j=1}^m [Z_{ij} - V_i^{(i)}(t_{ij})]^2, \quad (2.8)$$

$$V_i^{(i)}(t) = \mu_V^{(i)}(t) + \sum_{k=1}^M \frac{(i)}{ik} f_{ik}(t) \quad (2.9)$$

(5.5) $\int_{T_1}^T Q_V(t) + \frac{2}{W} \int_{T_1}^T G_V^*(t) dt \leq \int_{T_1}^T Q_V(t),$
(13).

$$\frac{2}{W} \int_{T_1}^T Q_V(t) + \frac{2}{W} \int_{T_1}^T G_V^*(t) dt \leq \{Q_V(t, t) + \frac{2}{W}\}_{+} \quad (2.10)$$

$$b_{Q_V} = \frac{2}{W}, \quad T = a_1, a_2, \dots, T = a_2 - a_1, \quad T_1 = a_1 + T/4, \quad a_2 - T/4 \quad (2003).$$

$$\frac{2}{W} = \frac{1}{T_1} \int_{T_1}^T \{Q_V(t) - G_V^*(t)\}_{+} dt \quad (2.11)$$

$$\frac{2}{W} > 0, \quad \frac{2}{W} = 0, \quad T/4 \quad (2003).$$

APPENDI B: APPROXIMATE ION COUNTING AND NOVEL IONS

$$S_i = V_i - V_{i-1} \quad (2.12)$$

$$(1) \quad S_i = V_i, \quad C > 0, \quad (2.13)$$

$$|S_i(t)| < C, \quad = 0, 1, 2, \quad (2.14)$$

$$V(t) < C, \quad (2.15)$$

$$b_{S,i} = b_{S,i}(n), \quad b_V = b_V(n), \quad h_V = h_V(n),$$

$$b_{Q_V} = b_{Q_V}(n), \quad S_i = S_i(n), \quad \mu_V = \mu_V(n), \quad (2.16)$$

$$G_V = G_V(t), \quad Q_V(t) = Q_V(t), \quad (2.17)$$

$$(2.1) \quad b_{S,i}, \quad b_S, \quad c_1, \quad c_2,$$

$$0 < c_1 < b_{S,i}/b_S, \quad b_{S,i}/b_S < c_2 < \infty.$$

$$(2.2) \quad m \rightarrow \infty, \quad b_S \rightarrow 0, \quad mb_S^2 \rightarrow \infty.$$

$$(2.3) \quad b_V \rightarrow 0, \quad b_{Q_V} \rightarrow 0, \quad nb_V^4 \rightarrow \infty, \quad nb_{Q_V}^4 \rightarrow \infty, \quad nb_V^6 < \infty, \quad nb_{Q_V}^6 < \infty.$$

$$(2.4) \quad h_V \rightarrow 0, \quad nh_V^6 \rightarrow \infty, \quad nh_V^8 < \infty.$$

$$(2.5) \quad n^{1/2} b_V m^{-1} < \infty, \quad n^{1/2} b_{Q_V} m^{-1} < \infty, \quad n^{1/2} h_V m^{-1} < \infty.$$

$$\{t_{ij}\}_{i=1, \dots, n, j=1, \dots, m} \quad (2.18)$$

$$i, \quad j = 1, \dots, m - 1, \quad t_{ij} < t_{ij+1}, \quad (2.19)$$

$$f_{ik} = \int_{T_1}^T f(t) dt = 1, \quad t \in T f(t) > 0, \quad (2.20)$$

$$t_{ij} < t_{ij+1}, \quad t_{ij} = F^{-1}(\frac{j-1}{m-1}), \quad F^{-1} = F(t) = \int_{a_1}^t f(s) ds, \quad (2.21)$$

$$t_{ij} < t_{ij+1}, \quad t_{ij} = c_1, \quad c_2, \quad 0 < c_1 < t_{ij}, \quad t \in T f(t) < N_i$$

$$i, \quad i \in T f(t) < c_2, \quad (2.22)$$

$$m \rightarrow \infty, \quad c_1, \quad c_2, \quad 0 < c_1 < i \frac{N_i}{m}, \quad i \frac{N_i}{m} < c_2 < \infty,$$

$$N_i = m, \quad i = 1, \dots, m - 1, \quad (2.23)$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sum_{j=1}^{m-1} |t_{ij} - t_{i,j-1}| = n = \sum_{j=1}^{m-1} (t_{ij} - t_{i,j-1}) : j = 2, \dots, m\},$$

(3) $n = O(m^{-1})$, $n, m \rightarrow \infty$.

$$\sum_{\substack{i,j \\ t \in T}} X_{ij} Z_{ij} < \infty, \quad \sum_j E X_{ij}^4 < \infty, \quad \sum_j E Z_{ij}^4 < \infty.$$

$$(f-g)(h) = [f, h] g, \quad f, g, h \in H,$$

$H = F \cap \mathcal{F}_2(H)$, $T_1, T_2 \in F$, $T_1 \hat{T}_2 = \sum_j (T_1 u_j, T_2 u_j)_H$, $T_F^2 = (T, T_F)$, $T_1, T_2, T \in F$, $\hat{T}_2 = \sum_j T_2 u_j$, $\{u_j : j = 1\}$, \mathbf{G}_V , G_V , (5) , G_V , (3) , $\mathcal{I}_i = \{j : j = i\}$, $\mathcal{T}' = \{i : \mathcal{I}_i = 1\}$, $\mathcal{I}_i = \{k : k \in \mathcal{I}_j\}$, $\mathbf{P}_j^V = \sum_{k \in \mathcal{I}_j} k$, $\mathbf{P}_j^V = \sum_{k \in \mathcal{I}_j} k$, H , $\{k : k \in \mathcal{I}_j\}$, $\{k : k \in \mathcal{I}_j\}$, j ,

$$\frac{V}{j} = \frac{1}{2} \sum_{l \in \mathcal{I}_j} \{l \neq j : l \notin \mathcal{I}_j\}, \quad (1)$$

$$\Lambda_j^V = \{z \in \mathcal{C} : z - j = \frac{V}{j}\}, \quad \mathcal{C} = \mathbb{R} \cup \mathbb{C}_{\infty},$$

$$\mathbf{R}_V(z) = (\mathbf{G}_V - zI)^{-1}, \quad \tilde{\mathbf{R}}_V(z) = (\mathbf{G}_V - zI)^{-1}.$$

$$A_j^V = \{ \mathbf{R}_V(z) : z \in \Lambda_j^V \} \quad (2)$$

$$M = M(n), \quad V_i(t) = \mu_V(t) + \sum_{m=1}^{M(n)} i m \cdot m(t), \quad (13), \quad \infty = \sum_{t \in T} (t) \quad (0), \quad \mu_V = \sum_{j=1}^n j, \quad M = M(n), \quad n \rightarrow \infty,$$

$$(5) \quad n = \sum_{j=1}^M (\frac{V}{j} A_j^V - j \infty) / (\bar{n} h_V^2 - A_j^V) \rightarrow 0, \quad M = M(n) \rightarrow \infty;$$

$$(6) \quad \sum_{j=1}^M j \infty = o(\bar{n} b_V, \bar{m}), \quad \sum_{j=1}^M j \infty = o(m).$$

$$(5) \quad (6) \quad j \in \mathcal{I}_j, \quad j = i, \quad j = k, \quad j = m = b_S^2 + (\bar{m} b_S)^{-1}, \quad V = \frac{V}{j} A_j^V - k \infty, \quad n =$$

$$(7) \quad E \{ \sum_{t \in T} V(t) - V^{(M)}(t) \}^2 = o(n), \quad V^{(M)}(t) = \mu_V(t) + \sum_{k=1}^M k \cdot k(t).$$

$$(8) \quad \sum_{k=1}^M (\frac{V}{k} A_k^V - k \infty) / (\bar{m} A_k^V) \rightarrow 0, \quad n \rightarrow \infty.$$

$$g(x, t) = t = t_{ij}, \quad t_1 = t_{ij_1}, \quad t_2 = t_{ij_2}, \quad i, j, j_1, \quad j_2, \quad g(x, t) = (X_{ij_1}, X_{ij_2}), \quad f(z, t) = f_2(z_1, z_2, t_1, t_2), \quad g_2(x_1, x_2, t_1, t_2), \quad g_2(z_{ij_1}, z_{ij_2}), \quad f_2(z_{ij_1}, z_{ij_2}), \quad g(t), f(t), t \in T, \quad g_2(t_1, t_2), f_2(t_1, t_2), t_1, t_2 \in T,$$

$$(1.1) \quad (d^2/dt^2)g(x, t) = (d^2/dt^2)f(z, t), \quad \Re \subset T, \quad$$

$$(1.2) \quad (d^2/dt_1^2 dt_2^2)g_2(x_1, x_2, t_1, t_2) = (d^2/dt_1^2 dt_2^2)f_2(z_1, z_2, t_1, t_2), \quad \Re^2 \subset T^2, \quad 1+2=2, 0-1, 2-2.$$

$$1: \mathbb{R} \rightarrow \mathbb{R}, \quad 2: \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$e^{iut} \int_1(u) du, \quad e^{iut} \int_2(t, s) = \int e^{iut+iws} \int_2(u, v) du dv.$$

$$(2.1) \quad \int_1^2 \frac{1}{2}(u) du < \infty, \quad \int_1^2(t) dt < \infty,$$

$$(2.2) \quad \int_2^2 \frac{1}{2}(u, v) du dv < \infty, \quad \int_2^2(t, s) dt ds < \infty.$$

APPENDIX C: PROOF

P

$$W = \sum_{i=1}^n S_i V, \quad \mathbb{E}^*(S_i V) = \sum_{j=1}^n R_j V, \quad (1)$$

$$R_j = R_j(1), \quad R_{ij} = R_{ij}(1), \quad R_{ij} = 0, \quad \mathbb{E}^*(R_j) = 0, \quad \mathbb{E}^*(R_j^2) < C_1, \quad (1)$$

$$\mathbb{E}^* \left(\sum_{t \in T} S(t) - S(t) \right) = O(b_S^2 + \frac{1}{\bar{m} b_S}), \quad (1)$$

$$S(t) = 0, 1, 2, \quad V = V(1), \quad (1), \quad (1),$$

$$\mathbb{E}^* \left(\sum_{t \in T} S(t) - S(t) \right) = O(b_S^2 + \frac{1}{\bar{m} b_S}), \quad (14).$$

$$V = \sum_{i=1}^n \{t_{ij}, Z_{ij}\}, \quad \{t_{ij}, Z_{ij}\}, \quad (2), \quad (3), \quad (4), \quad (5),$$

$$nk = \frac{k A_k^V}{\bar{n} h_V^2 - A_k^V}, \quad *_{nk} = \frac{k A_k^V}{m^2 - A_k^V}, \quad (1)$$

$$m = b_S^2 + (\bar{m} b_S)^{-1}, \quad \frac{V}{k} A_k^V, \quad (1), \quad (2).$$

Lemma C.1. $\sum_{t \in T} \mu_V(t) - \bar{\mu}_V(t) = O_p\left(\frac{1}{\bar{n} b_V}\right)$

$$G_V(s, t) - G_V(s, t) = O_p\left(\frac{1}{\bar{n} h_V^2}\right). \quad (1.2)$$

$\sum_{t \in T} |k(t)|^2 \leq L \cdot n \cdot k^2 + k^2 + 1, \quad k \geq 1, \quad t \in T.$

$$\begin{aligned} \sum_{t \in T} |k(t) - \bar{k}(t)| &= O_p(\sqrt{nk}) \\ |k - \bar{k}| &= O_p(\sqrt{nk}), \end{aligned} \quad (\text{C.3})$$

$nk \rightarrow 0, \quad n \rightarrow \infty, \quad k \geq 1, \quad nk \geq 1, \quad 1/k \leq M. \quad (\text{C.1}),$
 $O_p(\cdot) \leq (\text{C.3}), \quad \|\cdot\|_1 \leq 1/k \leq M. \quad (\text{C.2}),$

$$\sum_{t \in T} \frac{\hat{w}_V^2(t)}{W^2(t)} = O_p\left(1 + \left\{ \frac{1}{\bar{n}h_V^2}, \frac{1}{\bar{n}b_{QV}} \right\}\right). \quad (\text{C.4})$$

(1), (7), (1.1), (2.2),

$$\begin{aligned} \frac{1}{1/k/M} \sum_{k=1}^M ik - \bar{ik} &\xrightarrow{p} 0 \\ \sum_{t \in T} \left| \sum_{k=1}^M ik - k(t) - \sum_{k=1}^{\infty} ik - k(t) \right| &\xrightarrow{p} 0, \end{aligned} \quad (\text{C.5})$$

$M \geq 1, \quad M = M(n) \rightarrow \infty, \quad n \rightarrow \infty.$

Proof of Lemma C.1. By (C.2), (C.3), (C.5) and (2006), $\|\hat{w}_V^2\|_1$

$$\begin{aligned}
& + \sum_{t \in T} \left\{ \left| \sum_{k=1}^M ik^{-k} k(t) - \sum_{k=1}^{\infty} ik^{-k} k(t) \right| \right\} \\
& Q_{i1}(n) + Q_{i2}(n), \\
& Q_{i1}(n) \xrightarrow{P} 0, \quad Q_{i2}(n) \xrightarrow{P} 0. \quad (\text{5}), \quad Q_{i2}(n) \xrightarrow{P} 0 \quad (\text{6}), \quad (\text{7}), \\
& Q_{i2}(n) = O(\frac{1}{n}), \quad O(0) \quad (\text{8}), \\
& Q_{i1}(n) \xrightarrow{P} 0. \\
Q_{i1}(n) &= \sum_{t \in T} \left\{ \sum_{k=1}^M ik^{-k} k(t) \right. \\
& \quad \left. + \sum_{k=1}^M ik^{-k} k(t) - k(t) \right\}. \quad (\text{11}) \\
& \quad (\text{10}), \quad (\text{11})
\end{aligned}$$

$$C_1 \lim_{m \rightarrow \infty} \sum_{k=1}^M \frac{1}{k} + n \left\{ C_2 + \sum_{j=2}^m Z_{ij} (t_{ij} - t_{i,j-1}) \right\} \xrightarrow{P} 0.$$

$$\begin{aligned}
& O_p \left\{ \sum_{k=1}^M \frac{V}{k} A_k^{-1} V E^{-1} ik / \left(\frac{1}{m} \sum_{k=1}^M A_k^{-1} V \right) \right\}, \quad E \left\{ \sum_{k=1}^M \frac{V}{k} A_k^{-1} V E^{-1} ik / \right. \\
& \left. \left(\frac{1}{m} \sum_{k=1}^M A_k^{-1} V \right) \right\} = \sum_{k=1}^M \frac{V}{k} A_k^{-1} V / \left(\frac{1}{m} \sum_{k=1}^M A_k^{-1} V \right), \quad n, \\
& k \rightarrow 0, \quad 0, \quad 0, \\
& Q_{i1}(n) = O_p(\frac{1}{n}), \quad O_p(0), \quad i, \\
& (\text{18}), \quad t \in T, \quad V_i(t) - V_i(t) = O_p(\frac{1}{n} + \frac{2}{n}), \\
& O_p(0), \quad i, \quad n.
\end{aligned}$$

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