# DYNAMICALLY REGULARIZED HARMONY LEARNING OF GAUSSIAN MIXTURES

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### ABSTRACT

I i a e, a d, amicall, eg la i ed a m , lea i g D alg i m i éd f a ia mií e lea ig<sup>W</sup> i a fa i e fea e f b ada i e m del eleci a d c i le l a ame e lima i . S ecfi call, , de le famé fajeia i g-a g íam y lea i g, e i ili e i e a e age S a e i y fi e rei babili, e am lea a eg la i a i i em beigcılled by á cale faqıı e amy f qi a ia mi $_1$  'e i c ea i g f m  $0_1$  1 d, 'amicall. I, i dem  $\lambda$  a ed b  $\lambda$  e e e ime  $\lambda$  b  $\lambda$  y  $\lambda$  a ic a d e al-W ld da a a  $\lambda$  a  $\lambda$  e D alg i m c a  $\lambda$  k e lea lec eq mbe faq al a ia i le da á q, b l al bai te ma im m li<sub>k</sub>eli d M  $e_1$  ima  $f_1 e$ a ame e i l e aq al mi<sup>\*</sup>l e. M e e, l e D alg i mi calable a d ca be im leme led a big da a q. Keywords a ia mi e m del eleq i eg la i a i ma im m li eli d.

#### 1. INTRODUCTION

A are ible a d<sup>W</sup> efl taiticalt lf daa a at i a dif mai ce i g, fi te mit em del 1 a f di a licai i ma, blem, c a cl te i g a at i, image egmetai a d eec ec g ti . Am gt e ea licai , a ia mit e a d'idet ed a d e e al taitical lea i g ma d a ebee ed t deal t t i d fm del, c a te EM alg t m 2 a dt e ma d fm met 3. e, all amedt at e mbe f a ia cl te i a daa e i e-W Wee, i ma i ta ce ti e i fmati i tailable. e, te eleqti fa a i a ia mit e a di gi mit e. , te ge e al a ia mit em deli gi aq alt ac m d m deli g blem f b a aa e cm licaed a diffic taa 4.

ec e<sub>1</sub> i al<sup>W</sup> a f lig<sub>1</sub> i cm dmi- $_1$  em delig blem i lec a  $_1$  imal mbe  $k^*$  f a ia a 1 e cl 1e i 1eda a e ia e f 1ei - f mai, c di g a d 1ai 1ical eleqi c i e ia c aA ai e I f mai C i e i 5, a e ia I fee ce C i-1e i IC 6, Mi im m De c i 1 e g MD 7,a d Mi im m Me age e g MM 8. Wee, 1 ealida i g ce f i me d i c m 1ai all e ei e beca & e eed e ea 1 e e 1 e a ame e lea i gce a a la ge mbe f ible k.

Si ce 1990, me 1ai i cal lea i g a ac e a ea ed  $1 e_1$  i blem. Di ce la ce e 9 ad e e ible m Ma cai M 1eCal MCMC 10 a e ice i cal in leme 1ai f e f 1 i d f a ac e ice e 1 catic im la i e e 1 catic imla i ma d ge e all elie i 1e i e am li g a d a e e time-c mi g e e c d i di 1e a e ia m del ea c ba ed 1mi i g e a ia i al b d11.  $e_1 i d e e e e i e me a i g f i e mi e$ 12,  $13^{W}$  ic 1 d ce ce a i c m e i e me a i mi -<math>1 d e m d e e d c a a e e a e i a m del eleq i ca bemade ada <math>1e d i g a ama e lea i g i a im lfied MM m del eleq i ciei i g c c me a i i e c me a i mi.

Ale ai el, 1 e a e ia i g - a g am  $lea i g 14, 15, 16 a al ided a <math>\forall ai_1$  ical lea i g mec a i m<sub>1</sub> a ma e m del eleq i ada i el d i g a ame e lea i g. La al ead, bee im leme ied a ia mi<sub>1</sub> e lea i g a d e e al a m lea i g alg i m a e al bee e tabli ed f a ia mi<sub>1</sub> e 17, 18, 19. Al g e am 1 e abili, f ada i e m del elq i, i a ame e e timati a a table de iati f m<sub>1</sub> e M e timati <sup>W</sup> ic i c i te <sup>W</sup> i t e a ame e.

I de 1 le1 i de ia i blem, e c ide 1 e am lea i ga e M lea i g i a eglai ai 1 e m fa e age ega i e S a 1 f 1 e 1 e i babili, e am le i g1 c 1 1 e cale c m le i, fmi e m del. m1 i i1 f ie w e e ad amicall eg la i ed a m lea i g D alg i m f a ia mi e b addi g a S a 1 e g la i a i 1 e m bei g c 1 lled b a cale faq 1 e am lea i g. A e cale fac-1 i c ea ed d amicall f m 01 1, ed al-

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 $i^{W}$  e Ma,  $\iota$  e c e di g a  $\iota$  , el 86-10-62760609, Email  $^{W}$  ma ma ,  $_{i_{k}}$  .ed .c .

g  $i_1 m_1 a f m f m_1 e a m_y lea i g<sup>W</sup> <math>i_1$ ada i e m del eleq i  $i_1 \ i_2 e c e_1 i$  al ma im m li eli d lea i g  $i_1 a_1 e a da_1 i e m del eleq i a d$  $i_2 e M e_1 ima_i a e b_1 b_1 a i e da_1 a_1.$ 

e e<sub>1</sub> f<sub>1</sub> e a e i ga i ed a f l<sup>W</sup>. 5 e begi<sup>W</sup> i a b i ef de c i 1 i f<sub>1</sub> e a m , ea i g 1 em f a ia mi 1 e i Seqi 2. e <sup>W</sup> e e e 1 e de i a i a da al i f<sub>1</sub> e d amicall egla i ed a m , lea i g alg i m f á ia mi 1 e i Seqi 3. Seqi 4 c 1 ai 1 e e e i me 1 al e l b 1 y 1 e i c a d e al<sup>M</sup> ld da a e . i all<sup>W</sup> e c cl de b i en i Seqi 5.

### 2. BYY HARMONY LEARNING OF GAUSSIAN MIXTURES

e am y lea i g y tem de c ibe eac beai  $x \in X \subset \Re^n$  a d i c e di g i e e ee tai  $y \in Y \subset \Re^m$  iat e i g e f a e ia dec m i i ft e i t de i p(x,y) = p(x)p(y|x)a d q(x,y) = q(y)q(x|y); ic a e called a g mac i e a d i g mac i e, e eqi e i. i e a am le da a q  $D_x = \{x_t\}_{t=1}^N$  f m e a g be able ace, e a m y lea i g y tem i t y i g e t aqt e idde babilitic t q e f X i t e el fy f m ecif, i g all a eq f p(y|x), p(x), q(x|y) a d q(y) b ma imi i g e f  $\mathbb{N}^n$  i g a m y f q i al

$$H(p||q) = \int p(y|x)p(x)\ln[q(x|y)q(y)]dxdy. \quad 1$$

If b p(y|x) a d q(x|y) a e a ama ic, p(y|x) a d q(x|y) a e a ama ic, p(y|x)lea ig viemi calledi a e a i-dieqi al Acieq e i-Ácieq ef  $\chi$ . i e a am le da a e  $D_x =$  ${x_t}_{t=1}^N$ , e i-acieq e f, e am y lea i g y tem ca be ecfied a f l<sup>M</sup>. ei e e e tai  $\acute{y}$ i dicqei  $\curlyvee=\{1,2,\cdots,k\}$ i.e.," im=1," ile  $\lambda$ e be ai xic  $\lambda$ i f m a a ia mi $\lambda$ e di lib li . O le i g ace, e le  $q(y=j) = \pi_j \ge 0$ <sup>W</sup> i  $\sum_{j=1}^{k} \pi_j = 1$ . i i a i babili di ib i f a ia cl  $\iota$ e f $\iota$  e mi $\iota$  e. O  $\iota$  e a g ace, p(x) i a la e babili, de i, f a i df f a ia mi  $_{1}$  ef m<sup>W</sup> ic  $D_{x}$  a ége e a ed. M e e, i  $\iota$  e i g a  $, q(x|y=j) = q(x|m_j, \Sigma_j)$  i a med  $\iota$  be a a ia de i f a W i mea ea  $m_j$  a d'e c a ia ce ma i  $\Sigma_j^{W}$  ile i te a g a , p(y = j|x) i c ∖ qed de ∖ e ą e ia i ci le b, \ e f l<sup>W</sup> i g a ame ic f m,

$$p(y=j|x) = \frac{\pi_j q(x|m_j, \Sigma_j)}{q(x|\Theta_k)}, \qquad 2$$

$$q(x|\Theta_k) = \sum_{j=1}^k \pi_j q(x|m_j, \Sigma_j), \qquad 3$$

<sup>w</sup>  $e \in \Theta_k = {\pi_j, m_j, \Sigma_j}_{j=1}^k$  a  $d q(x|\Theta_k)$  i  $\lambda$  a aia mi $\lambda$  em del $\lambda$  a<sup>w</sup> ill a ima e $\lambda$  e la e  $\lambda p(x)$  ia  $\lambda$  e a m  $\lambda$  lea i g  $\lambda$  e lea i g  $\lambda$  em.  $\lambda$  all $\lambda$  e e c m e  $\lambda$  i  $\lambda$  E  $\lambda$  1<sup>w</sup>, e a e

 $H(p||q) = E_{p(x)} \left[ \sum_{j=1}^{k} h_j(X) \ln[\pi_j q(X|m_j, \Sigma_j)] \right], \quad 4$ 

₩ ee

$$h_j(X) = \frac{\pi_j q(X|m_j, \Sigma_j)}{\sum_{i=1}^k \pi_i q(X|m_j, \Sigma_j)}.$$
 5

a i, H(p||q) i e e eqai faf qi fe a d m a iable X b eq p(x). 5 i e am le da a q  $D_x$ , e ge a e ima e fH(p||q) called a m y f ci, a f ll.

$$J(\Theta_k) = \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{k} h_j(x_t) \ln[\pi_j q(x_t | m_j, \Sigma_j)].$$
 6

Acc di gi i ei e qicala de e ime i al e l i i-acieq e fi e am y lea i g y iem f a ia mi e 20, 17, 18, 19, i e ma imi ai f  $J(\Theta_k)$  i ca able f ma i g m del eleqi ada i el di g a amqe lea i g<sup>w</sup> e i e aq al a ia cli e a e e a qedi a cei ai deg ee. q i i i e c e  $k_1$  be la ge i a i e mbe  $k^*$  f aq al a ia cl i e a m le da a, i e ma imi ai fi e a m y f qi ca ma e  $k^*$  a ia i macieq a al e a d im la e l'elimi qe  $k-k^*$  e a e. W e e, a W e me i ed e i l, i e igi al a m y lea i g ffe f mi c i e e g la i ai meca i mi a f mi e a m y lea i gi e M lea i g c i a da i e m del eleqi a d c i e i a amqe e i mai ca be made im la e l.

### 3. DYNAMICALLY REGULARIZED HARMONY LEARNING ALGORITHM

#### 3.1. The Dynamic Regularization Mechanism

Acc di g<sub>1</sub> 21,  $J(\Theta_k)$  ca be di ided i  $V_1^W$  a<sub>1</sub>,

$$J(\Theta_k) = L(\Theta_k) - O_N(p(y|x)), \qquad 7$$

<sup>W</sup> e e<sub>1</sub> e<sup>n</sup> 1 a<sub>1</sub> i 11 e l g-li<sub>k</sub>eli df qi , i.e.,

$$L(\Theta_k) = \frac{1}{N} \sum_{t=1}^{N} \ln(\sum_{j=1}^{k} (\pi_j q(x_t | m_j, \Sigma_j))),$$
 8

<sup>W</sup> ile<sub>1</sub> e ec di 1 e a e age S a e 1 f<sub>1</sub> e -1 e i babili<sub>y</sub> p(y|x) e 1 e am le da a e  $\mathcal{D} = \{x_t\}_{t=1}^N$ ,

$$O_N(p(y|x)) = -\frac{1}{N} \sum_{t=1}^N \sum_{j=1}^k p(j|x_t) \ln p(j|x_t).$$
 9

Acc di g<sub>1</sub> E . 7, if  $-O_N(p(y|x))$  i de da a egla i a i 1 e m, i e a m y lea i g, i.e., ma imi i g  $J(\Theta_k)$ , i a eg la i ed M lea i de i de i de da e eq bee i e i ga ed i 22, 23 b, cali g e eg la i a i i e m i a mall i e m be. We e, i ce e ee i e eg la i a i cale c 1 a i e ca e f i e a m y lea i g, i e e i e i ga i a ffe f m i c i e i a ama e e i ma i.

 $O_1 e_1 e_2 a_1 d_2 f_1 m E_2 T^{W} e_1 a_2 e_2$ 

$$L(\Theta_k) = J(\Theta_k) + O_N(p(y|x)), \qquad 10$$

<sup>w</sup> ic i dcae  $\iota$  a  $\iota$  e M lea i g i a eg la i ed a m v lea i  $g^{W}$  i  $O_N(p(y|x))$  a  $\iota$  e eg la i a i  $\iota$  e m. c  $\iota$  l  $\iota$  e eg la i a i , a cale fac  $\lambda (\geq 0)$ i i  $\iota$  d ced,

$$L_{\lambda}(\Theta_k) = J(\Theta_k) + \lambda O_N(p(y|x)).$$
 11

If  $\lambda = 0$ ,  $L_{\lambda}(\Theta_k) = J(\Theta_k)$  i  $\lambda = 0$ ,  $L_{\lambda}(\Theta_k) = J(\Theta_k)$  i  $L_{\lambda}(\Theta_k)$  i  $\lambda = 1$  g-li eli df qi f $\lambda = 1$ ,  $L_{\lambda}(\Theta_k)$  i  $\lambda = 1$  g-li eli df qi f $\lambda = 1$ ,  $L_{\lambda}(\Theta_k)$  i  $\lambda = 1$  g-li eli df qi f $\lambda = 1$ ,  $L_{\lambda}(\Theta_k)$  i  $\lambda = 1$  g-li eli g f 0, 1, ma imii g  $L_{\lambda}(\Theta_k)$  c a ge f m $\lambda = 1$  g m g lea i g  $\lambda = 1$  g m lea i g eli e ada  $\lambda = 1$  g m del eleq i  $a_{\lambda} = 0$  i g f  $\lambda = 1$ i a eli  $\lambda = 1$  e e i

lea i'g tage a dt e M e tima i at e f al lea i g tage.

### 3.2. The Fixed-point Learning Algorithm

At eac a e f<sub>1</sub> e d, amicall, eg la i ed a m, lea i g<sup>W</sup> i a ecfic  $\lambda$ , e c 1 c, a fi ed- i l alg i m l ma imi e  $L_{\lambda}(\Theta_k)$ .

c e ie ce, e tili et e fima e e etai f  $\pi_j$ , i.e.,  $\pi_j = e^{\beta_j} / \sum_{i=1}^K e^{\beta_i}$ ,  $j = 1, \dots, k$ , e e  $\beta_j \in (-\infty, +\infty)$ ,  $j = 1, \dots, k$ . et i gt e de i ai e f  $L_{\lambda}(\Theta_k)^{W}$  i e eqt  $\beta_j$ ,  $m_j$  a d  $\Sigma_j$ , e eqi el, be e W e get e f ll i g e ed- it ie ai e lea i g alg i m

$$\hat{\pi}_{j} = \frac{\sum_{t=1}^{N} p(j|x_{t})\gamma_{j}(t)}{\sum_{t=1}^{K} \sum_{i=1}^{k} p(i|x_{t})\gamma_{i}(t)}; \qquad 12$$

$$\hat{m}_{j} = \frac{\sum_{t=1}^{N} p(j|x_{t})\gamma_{j}(t)x_{t}}{\sum_{t=1}^{N} p(j|x_{t})\gamma_{j}(t)};$$
13

$$\hat{\Sigma}_{j} = \frac{\sum_{t=1}^{N} p(j|x_{t})\gamma_{j}(t)(x_{t} - \hat{m}_{j})(x_{t} - \hat{m}_{j})^{T}}{\sum_{t=1}^{N} p(j|x_{t})\gamma_{j}(t)}, \quad 14$$

₩ ee

$$\gamma_{i}(t) = 1 - \sum_{l=1}^{k} (p(l|x_{t}) - \delta_{il}) \ln \pi_{l} p(x_{t}|m_{l}, \Sigma_{l}) + \lambda \sum_{l=1}^{k} (p(l|x_{t}) - \delta_{il}) \ln p(l|x_{t}),$$
 15

 $^{W}$  ee $\delta_{ij}$ ile ecefai.

I c m ai <sup>W</sup> į <sup>k</sup><sub>1</sub> c c c i al EM alg į m f a ia mi i c 2, ed fi ed-ii lea i galg į m diffe l ai ea gme i gi e m  $\gamma_j(t)$ . Į ca be ea il c fi ed a<sup>W</sup> e  $\lambda = 1, \gamma_j(t) = 1, i$  e fi edii lea i galg į m i i c EM alg į m a d<sup>V</sup> e  $\lambda = 0, i$  e fi ed-ii lea i galg į m e i i e igi al fi ed-ii lea i galg į m f ma imi i g i e a m y f qi  $J(\Theta_k)$ .

Aq ali,  $\gamma_j(t)$  im leme  $\iota$  a i al e ali ed c m qili e lea i g C mec a i m 24 - 25  $\iota$  a m del eleq i ca be made ada  $\iota$  i el d i g a amq e lea i g. At  $\iota$  e ea  $\iota$  lea i g  $\iota$  age,  $\gamma_j(t) < 0$  ma a e. Acc di g  $\iota$  E. 15,  $\iota$  e mea eq f  $j_{i}$  a ia<sup>W</sup> ill m e W a f m  $x_t$ . Q e i e, if  $\gamma_j(t) > 0$ ,  $\iota$  e mea eq f  $\iota$  e  $j_{i}$  a ia<sup>W</sup> ill be at aq ed  $\iota$   $x_t$ . S, f  $x_t$ , a ia W i  $\gamma_j(t) > 0$  a e i e.

W e e, l e fi ed- i lea i galg i m ca lg aaleel e i i e defi i e e feac c a ia ce ma i d i gleieai i ce  $\gamma_j(t)$  ma be egai e. I de le c mel i blem, e el e EM dae le fle c a ia ce ma i e, i.e., f ci g all  $\gamma_j(t) = 1$  i E. 14, i li cefic ca e. I faq, li im lfi ca i i a licable a d eficiel i cel e c meli fi adali e m del eleq i i mai la mig mea equad c lled by lemi i g ji.

#### **3.3.** The Dynamic Evolution of $\lambda$

$$h_{\pi}(T) = |\frac{H_{\pi}(T) - H_{\pi}(T-1)}{H_{\pi}(T)}|, \qquad 16$$

a a i dica fm del eleqi . T i time, i.e., te mbe fiteati  $\mathcal{W}$  le lea i g ce i di ided i t  $\mathcal{W}$  lea i g tage acc di g agi e t e ld  $\varepsilon_1(>0)$ ft i dica . a i, if  $h_{\pi}(T) > \varepsilon_1$ ,  $\lambda(T)$  i cea e a 
$$\lambda(T) = \begin{cases} \lambda_0 * \eta_1^T, & \text{if } h_\pi(T) > \varepsilon_1; \\ \lambda_0 * (\frac{\eta_1}{\eta_2})^{T^*} \eta_2^T, & \text{if } h_\pi(T) \le \varepsilon_1, \end{cases}$$
 17

<sup>W</sup> ee  $\lambda_0$  beiga e, mall ii ec iai i i ial al e f $\lambda$ ,  $\eta_1$ ,  $\eta_2$  a e<sup>W</sup><sub>1</sub> i ec iai<sup>W</sup> i c i ai i a 1 <  $\eta_1$  <  $\eta_2$ , a d  $T^*$  i e i g i c i a  $h_{\pi}(T^*) > h_0$  a d  $h_{\pi}(T^*+1) \le h_0$ . for i illicit e ge.

**3.4.** The Complete DRHL Algorithm



а





| Паае            | D         |       | $CEM^2$   |              |
|-----------------|-----------|-------|-----------|--------------|
| Daaq            | CMS e e ç | ۱ ime | CMS e e ç | <i>\</i> ime |
| $\mathcal{S}_1$ | 100       | 526   | 84        | 11290        |
| $\mathcal{S}_2$ | 100       | 856   | 56        | 1825         |
| $\mathcal{S}_3$ | 100       | 145   | 72        | 4317         |
| $\mathcal{S}_4$ | 96        | 460   | 56        | 554          |

**Table 2**. ecmai  $f_1 e D$  a d  $CEM^2$  alg  $i_1 m$  m del elegi a d  $i_1$ ime.

Table 3.e c m a i $f_1 e D$ a d  $CEM^2$  alg -i ma ame e i ima iaccacc

| Da a e                     | D      | $\operatorname{CEM}^2$ |
|----------------------------|--------|------------------------|
| $\overline{\mathcal{S}_1}$ | 0.0204 | 0.0204                 |
| $\overline{\mathcal{S}_2}$ | 0.0171 | 0.0172                 |
| $\mathcal{S}_3$            | 0.0363 | 0.0363                 |
| $\mathcal{S}_4$            | 0.0308 | 0.0715                 |

 $ige_1a_1ef_1eD$  alg i m.

# 4.2. Unsupervised Classifications of Iris and Wine Data

5 ef 1 e a  $l_1$  e D alg  $i_1$  m<sub>1</sub> 1 e e i ed ca ficai f<sub>1</sub> e I i a d 5 i e da af m UCI Mac i e ea i g e  $i_1$  28. e I i da a e c 1 ai 1 ee cla e, I i e i c 1 , I i i gi i ca a d I i Se a, a d eac cla c  $i_1$  f50 am le. Eac am lei 4-dime i al eq mea i g e la m 1 g. I e e ime 1 e I i da a,<sup>w</sup> e e e e i i i al e f k a 6 a d ei i i al e f e e a ame e a i im lai e e ime 1 e e all , e D alg i m ed a  $k^* = 3^w$  i e i imal cla ficai acc ac 96.7 O l fi ef m 150 am le a emi cla fied. <sup>w</sup> e e i i i ble a e D alg i m c e g k k = 2. Si ce e a et I i b-cla e<sup>w</sup> ic a e 1 g e la ed, melie a e al acce k = 2.

e 5 i e da a e i 13-dime i al a d c i 1 f 178 am le  $f_1$  e e i e . I i ca e, e e c i da a e b i e i ci al c m e a al i CA dime i ed qi i c i e 29 a d c e l e fi i c d q e i ci le c m e c e D alg i mi c d q e i c e c e ed da i i e i i al al e f k a 6. E e i me al e l dem i a e a c a c f c la finca i ca e ac a 98.3 O l e e am le a e mi c la fin ed .

#### 5. CONCLUSIONS

<u>5</u>e a e i e i ga e di e e la i i be<sup>w</sup> e e i e a n lea i g a d l e M lea i g a d b idged l em -, f igáeg laia, i lem leaeage Sa e 1 ,  $\iota e \iota e i$  babili, e am le. a ed c a' eg-la i a i mec a i m, e c  $\iota$  c  $\iota$  e d, amicall, egla i ed a m y lea i g D f a ia mi  $_{1}$  e. c  $_{1}$  lli g e cale faq f i eg la i a i  $_{1}$  e m d amicall i c ea e f m 0  $_{1}$  1,  $_{1}$  e D alg i m laı fmi e am v lea ig<sup>y</sup>i a ca abili<sub>v</sub> f ada i e m del elegi , a'd e g ad all i a f m i ιec eli alma im mligeli dlea íigι bai a c ilel a ama e elima i<sup>\*</sup>. M e elle D alg i m i calable a d ca be ed a big da a  $e_i^{W}$  i celai da a mmaia i lec i e. E e imelal e l dem  $1 a e_1 a$ ,  $b_1 y 1 e_1 c_2 d e_2 M^2$  ld da  $a e_1$ , 1 e D alg  $i m c_2 1$ ,  $i g e_1 e_1 e_2 e_3$  mbe faq al a ia i a da a 'a, b l al b ai l e M elimae fle a ame e i le mi le.

# Acknowledgments.

i<sup>w w</sup>a 1ed by 1 e a al Sciece da i fCiafi a 1, 61171138 a d 60771061.

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