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# Parameter estimation of Poisson mixture with automated model selection through BYY harmony learning

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## ABSTRACT

Finite mixture is widely used in the fields of information processing and data analysis. However, its model selection, i.e., the selection of components in the mixture for a given sample data set, has been still a rather difficult task. Recently, the Bayesian Ying–Yang (BYY) harmony learning has provided a new approach to the Gaussian mixture modeling with a favorite feature that model selection can be made automatically during parameter learning. In this paper, based on the same BYY harmony learning framework for finite mixture, we propose an adaptive gradient BYY learning algorithm for Poisson mixture with automated model selection. It is demonstrated well by the simulation experiments that this adaptive gradient BYY learning algorithm can automatically determine the number of actual Poisson components for a sample data set, with a good estimation of the parameters in the original or true mixture where the components are separated in a certain degree. Moreover, the adaptive gradient BYY learning algorithm is successfully applied to texture classification.

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## 1. Introduction

As a powerful probabilistic model, finite mixture distribution has been adopted extensively in a wide variety of practical situations where data can be viewed as arising from two or more populations linearly mixed in certain proportions (e.g. [1–3]). In fact, there are a variety of finite mixtures. Among them, Gaussian mixture is the most well known and frequently used. Clearly, the sample data subject to a Gaussian mixture should be continuous. However, there are many available discrete data which can be regarded being generated from a finite mixture model. Certainly, we can transform these discrete data into continuous ones via some appropriate techniques and still use the Gaussian mixture model to analyze them, e.g. like the correspondence analysis of categorical data. But such a transformation approach has serious limitations because some useful information can be lost during the transformation. Moreover, the Gaussian or normal assumption may not be appropriate for some practical problems, especially in the cases of count data. For these reasons, more attention is being focused on the finite mixtures whose components are not Gaussian densities. Actually, Poisson mixture is a typical non-Gaussian finite mixture with a variety of practical applications such as biological and medical data modeling (e.g. [4,5]), analysis of user accesses to web pages [6], text mining [7], shopper classification [8]

and gray level texture classification [9]. Theoretically, Poisson mixtures are identifiable [10] and owns a great number of properties (refer to the recent review [11]).

In the conventional finite mixture modeling or parameter learning, in particular for Poisson mixture, it is usually assumed that the number  $k$  of components in the mixture is pre-known. In this situation, several statistical or unsupervised learning methods have been established for parameter estimation or learning. The most classical method of this kind may be the method of moments (e.g. [12,13]). But it can only solve the problem of parameter estimation for the mixture with two components or a number of simple components. Clearly, the maximum likelihood principle can be applied to the parameter estimation of finite mixture in general, which led to the EM algorithm for finite mixture [14,15]. Although the EM algorithm has certain good convergence behaviors [16–18], it still has some weaknesses or limitations. Essentially, the EM algorithm is a local searching approach, thus “bad” initialization can make it get trapped in a local solution. Moreover, the Bayesian inference can also be utilized to solve the parameter estimation of finite mixture [3], but it is rather difficult to set up a set of reasonable prior distributions for the parameters in the mixture. On the other hand, the parameter estimation problem of finite mixture can be solved with the minimum distance principle with the help of defining certain distance measure between the underlying (or actual) and the estimated distributions (e.g. [2,19]). Although many such distances have been established, it is usually rather difficult to implement the minimum distance principle for the parameter estimation of finite mixture efficiently.

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When the number of components is not known in advance, the finite mixture modeling becomes complicated and difficult because the selection of an appropriate number of components must be made jointly with the estimation of the parameters [20]. Since the number of components is just a scale of the finite mixture model, its selection for the mixture to model a sample data set is usually referred to as the model selection. Certainly, as a typical class of finite mixtures, Poisson mixtures face the same compound modeling problem of parameter estimation or learning and model selection. Although this problem might be solved by choosing a best number  $k^*$  of components as the clusters in the sample data set via one of information, coding and statistical selection criteria such as Akaike's Information Criterion [21] or its extensions (e.g. [22,23]), MDL [24], MML [25], likelihood ratio test (LRT) [26] and the Bootstrapping methods [27,28], the process of evaluating a criterion incurs a large computational cost since we need to repeat the entire parameters learning process at a large number of different values of  $k$ . Moreover, all the existing theoretical selection criteria have their limitations and often lead to a wrong result.

Since 1990s, there have appeared some new approaches to solve this compound mixture modeling problem. One approach was to use a kind of stochastic simulation to infer the optimal mixture model. The two typical implementations are the methods of Dirichlet processes [29] and reversible jump Markov chain Monte Carlo (RJMCMC) [30]. However, these stochastic simulation methods generally require a large number of samples through different sampling rules. Another approach was the unsupervised learning [31] on finite mixture which introduces certain competitive learning mechanism into the EM algorithm such that the model selection can be made during the parameter learning by annihilating the components with very small mixing proportions during the parameter learning via the MML principle. As a matter of fact, our proposed approach in the current paper follows a similar rule on the model selection and thus

distribution can be defined via a multivariate reduction technique. For example, in the case of  $q = 2$ , we can use three independent Poisson random variables  $Y_i$ , with parameters  $\theta_i$ , for  $i \in S = \{1, 2, 0\}$ , and obtain the following representation formulae respectively for  $X_1, X_2$ :

$$X_1 = Y_1 + Y_0,$$

$$X_2 = Y_2 + Y_0.$$

In this situation, matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix},$$

and  $Y = (Y_1, Y_2, Y_0)^T$ . Particularly, the probability distribution or function of  $X = [X_1, X_2]^T$  at each point  $x = (x_1, x_2)^T$  is given as follows:

$$p(x_1, x_2 | \theta_1, \theta_2, \theta_0) = e^{-(\theta_1 + \theta_2 + \theta_0)} \frac{\theta_1^{x_1} \theta_2^{x_2}}{x_1! x_2!} \sum_{i=0}^{\min(x_1, x_2)} \binom{x_1}{i} \binom{x_2}{i} i! \left(\frac{\theta_0}{\theta_1 \theta_2}\right)^i. \quad (2)$$

Clearly,  $Y_0$  acts as a correlation term between  $X_1$  and  $X_2$ . Moreover, the correlation can be seen from the expressions of mean vector and covariance matrix of the random vector  $X$ , which can be computed as follows:

$$E(X) = \mathbf{A}M, \quad (3)$$

$$\text{Var}(X) = \mathbf{A}\Sigma\mathbf{A}^T, \quad (4)$$

where

$$M = E(Y) = (\theta_1, \theta_2, \dots, \theta_m)^T$$

and  $\Sigma$  is the covariance matrix of  $Y$  given by

$$\Sigma = E[(Y - E(Y))(Y - E(Y))^T] = E[(Y - M)(Y - M)^T] = \text{diag}[\theta_1, \theta_2, \dots, \theta_m].$$

For practical applications with the multivariate Poisson model, we generally assume that either there is a common correlation-generating term for any two random variables or simply all the variables are independent in the random vector. For the former correlation case, we let  $X = (X_1, X_2, \dots, X_m)^T$  be constructed from  $Y = (Y_0, Y_1, \dots, Y_m)^T$  via the simple relations:  $X_i = Y_i + Y_0 (i = 1, \dots, m)$ , where each  $Y_i$  is an independent Poisson random variable with parameter  $\theta_i$ . In this way, the probability distribution or function of  $X$  at each point  $x = (x_1, x_2, \dots, x_m)^T$  can be expressed as follows:

$$p(x_1, x_2, \dots, x_m | \theta_0, \theta_1, \dots, \theta_m) = \exp\left(-\sum_{i=0}^m \theta_i\right) \prod_{i=1}^m \frac{\theta_i^{x_i}}{x_i!} \sum_{l=0}^s \prod_{j=1}^m \binom{x_j}{l} l! \left(\frac{\theta_0}{\prod_{k=1}^m \theta_k}\right)^l, \quad (5)$$

where  $s = \min(x_1, x_2, \dots, x_m)$ .

For the latter independent case, supposing that all the random variables  $X_i$  are independent Poisson random variables with parameters  $\theta_i$ , we can easily get the probability distribution or function of  $X = (X_1, X_2, \dots, X_m)^T$  at each point  $x = (x_1, x_2, \dots, x_m)^T$  as follows:

$$p(x_1, x_2, \dots, x_m | \theta_1, \theta_2, \dots, \theta_m) = \prod_{i=1}^m \frac{\theta_i^{x_i}}{x_i!} e^{-\theta_i}. \quad (6)$$

## 2.2. Poisson mixture

In many practical applications, the observed data can be considered being generated from a number of components that are linearly mixed in certain proportions. That is, the observed data are subject to a finite mixture distribution. The major task is then to solve the compound mixture modeling problem of model selection and parameter estimation, i.e., to determine the number of components and estimate the parameters of the component distributions as well as the mixing proportions, only with a set of sample data. Theoretically, we consider the following finite mixture model:

$$q(x|\Theta_k) = \sum_{j=1}^k \alpha_j q(x|\theta_j), \quad (7)$$

where  $q(x|\theta_j)$  are component probability densities or distributions with parameters  $\theta_j$ ,  $k$  is the number of components in the mixture,  $x$  denotes the variable or variable vector, and  $\alpha_j \geq 0$  are mixing proportions of the components with the constraint that  $\sum_{j=1}^k \alpha_j = 1$ . For clarity, we let  $\Theta_k = \{\alpha_j, \theta_j\}_{j=1}^k$  be the set of all parameters in the mixture model.

If all  $q(x|\theta_j)$  in Eq. (7) are Poisson probability distributions, the finite mixture is called a Poisson mixture. As a typical class of finite mixtures, Poisson mixtures are not only important in statistics, but also widely used in practical applications. As discussed in the previous section, many efforts have been made on the model selection and parameter estimation of Poisson mixture. As a matter of fact, the EM algorithm is probably the most frequently used method to estimate the parameters of the Poisson mixture with a sample data set [14,15]. However, the EM algorithm is constructed under a framework of maximum likelihood and thus is unable to make model selection for Poisson mixture only with a set of sample data. In the following, based on the BYY harmony learning, we will construct a BYY harmony learning algorithm for Poisson mixture to make model selection automatic during parameter learning.

## 3. Adaptive gradient BYY learning algorithm

In this section, we further introduce the BI-architecture of the BYY learning system on which the harmony learning turns into the parameter learning with automated model selection on the finite mixture model [34,36], and then derive the adaptive gradient BYY learning algorithm for Poisson mixture.

### 3.1. BYY learning system and harmony function for Poisson mixtures

A BYY system describes each observation  $x \in \mathcal{X} \subset \mathfrak{R}^n$  and its corresponding inner representation  $y \in \mathcal{Y} \subset \mathfrak{R}^m$  via the two types of Bayesian decomposition of the joint density:  $p(x, y) = p(x)p(y|x)$  and  $q(x, y) = q(y)q(x|y)$ , which are called Yang machine and Ying machine, respectively. Given a data set  $D_x = \{x_t\}_{t=1}^N$  from the Yang or observable space, the goal of harmony learning on a BYY learning system is to extract the hidden probabilistic structure of  $x$  with the help of  $y$  from specifying all aspects of  $p(y|x)$ ,  $p(x)$ ,  $q(x|y)$  and  $q(y)$  via a harmony learning principle implemented by maximizing the functional

$$H(p||q) = \int p(y|x)p(x) \ln[q(x|y)q(y)] dx dy, \quad (8)$$

which is essentially equivalent to minimizing the Kullback–Leibler divergence between the Yang and Ying machines, i.e.,  $p(x, y)$  and  $q(x, y)$ , because

$$KL(p||q) = \int p(y|x)p(x) \ln \frac{p(y|x)p(x)}{q(x|y)q(y)} dx dy = -H(p||q) - H(p), \quad (9)$$

where  $H(p)$  is the entropy of  $p(x, y)$  and invariant to  $q(x, y)$ .

If both  $p(y|x)$  and  $q(x|y)$  are parametric, i.e., from a family of probability densities with parameter  $\theta$ , the BYY learning system is said to have a BI-directional architecture (BI-architecture). For the Poisson mixture model with a given sample set  $D_x = \{x_t\}_{t=1}^N$ , we can utilize the following specific BI-architecture of the BYY learning system. The inner representation  $y$  is discrete in  $\mathcal{Y} = \{1, 2, \dots, k\}$  (i.e., with  $m = 1$ ), and the observation  $x$  is also discrete from a Poisson mixture distribution. On the Ying space, we let  $q(y = j) = \alpha_j \geq 0$  with  $\sum_{j=1}^k \alpha_j = 1$ . On the Yang space, we suppose that  $p(x)$  is a blind Poisson mixture probability distribution, with a set of sample data  $D_x$  being generated from it. Moreover, in the Ying path, we let each  $q(x|y = j) = q(x|\theta_j)$  be a Poisson probability distribution with parameter  $\theta_j$  consisting of all its parameters, while the Yang path is constructed under the Bayesian principle by the following parametric form:

$$p(y = j|x) = \frac{\alpha_j q(x|\theta_j)}{q(x|\Theta_k)}, \quad q(x|\Theta_k) = \sum_{j=1}^k \alpha_j q(x|\theta_j), \quad (10)$$

where  $\Theta_k = \{\alpha_j, \theta_j\}_{j=1}^k$  and  $q(x|\Theta_k)$  is just a Poisson mixture that will approximate the true Poisson mixture  $p(x)$  hidden in the sample data  $D_x$  via the harmony learning on the BYY learning system.

With all these component densities in Eq. (8), we have

$$H(p||q) = E_{p(x)} \left[ \sum_{j=1}^k \frac{\alpha_j q(X|\theta_j)}{\sum_{i=1}^k \alpha_i q(X|\theta_i)} \ln[\alpha_j q(X|\theta_j)] \right], \quad (11)$$

that is, it becomes the expectation of a random variable  $\sum_{j=1}^k ((\alpha_j q(X|\theta_j)) / (\sum_{i=1}^k \alpha_i q(X|\theta_i))) \ln[\alpha_j q(X|\theta_j)]$ , where  $X$  is just the random variable (or vector) subject to  $p(x)$ . Based on the given sample data set  $D_x$ , we get an estimate of  $H(p||q)$  as the following harmony function for Poisson mixtures with the parameter set  $\Theta_k$ :

$$J(\Theta_k) = \frac{1}{N} \sum_{t=1}^N \sum_{j=1}^k \frac{\alpha_j q(x_t|\theta_j)}{\sum_{i=1}^k \alpha_i q(x_t|\theta_i)} \ln[\alpha_j q(x_t|\theta_j)]. \quad (12)$$

Based on Eqs. (8), (11) and (12), we actually have a new derivation of the harmony function  $J(\Theta_k)$  in finite mixture. With help of probability theory and statistics, this derivation is more reasonable and clearer than the previous derivation in [34,36,37].

According to the BYY harmony learning principle [34,35], the maximization of the harmony function  $J(\Theta_k)$  is able to make model selection automatic during parameter learning for Poisson mixture with a sample data set in which the number  $N$  of sample points is large enough and the actual components are separated in a certain degree. That is, in such a situation, as long as we set  $k$  to be larger than the number  $k^*$  of actual Poissons in the sample data, it can make  $k^*$  Poissons from the estimated mixture match the actual Poissons, respectively, and force the mixing proportions of the other  $k - k^*$  extra Poissons to attenuate to zero. In order to do so, we will construct an adaptive gradient BYY learning algorithm to search the maximum of  $J(\Theta_k)$  in the next subsection.

### 3.2. Derivation of the adaptive gradient BYY learning rule

For convenience of derivation of  $J(\Theta_k)$ , we introduce a group of intermediate variables  $U_j(x) = \alpha_j q(x|\theta_j)$  for  $j = 1, 2, \dots, k$ , as we previously did in [37]. In this way,  $J(\Theta_k)$  takes a simple and structural form:

$$J(\Theta_k) = \frac{1}{N} \sum_{t=1}^N J_t(\Theta_k), \quad J_t(\Theta_k) = \sum_{j=1}^k \frac{U_j(x_t)}{\sum_{i=1}^k U_i(x_t)} \ln U_j(x_t). \quad (13)$$

Moreover, in order to get rid of the constrains on  $\alpha_j$ , we utilize the following so-called softmax representation:

$$\alpha_j = \frac{e^{\beta_j}}{\sum_{i=1}^k e^{\beta_i}}, \quad j = 1, 2, \dots, k, \quad (14)$$

where  $-\infty < \beta_1, \dots, \beta_k < +\infty$ .

With the above preparations, we can get the derivatives of  $J(\Theta_k)$  with respect to  $\beta_j$  and  $\theta_j$  at sample point  $x_t$  as follows:

$$\begin{aligned} \frac{\partial J_t(\Theta_k)}{\partial \beta_j} &= \sum_{i=1}^k \frac{\partial J_t(\Theta_k)}{\partial U_i(x_t)} \frac{\partial U_i(x_t)}{\partial \beta_j} \\ &= \frac{1}{q(x_t|\Theta_k)} \sum_{i=1}^k \left[ 1 - \sum_{l=1}^k (p(l|x_t) - \delta_{il}) \ln U_l(x_t) \right] \\ &\quad \times (\delta_{ij} - \alpha_j) U_i(x_t), \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{\partial J_t(\Theta_k)}{\partial \theta_j} &= \sum_{i=1}^k \frac{\partial J_t(\Theta_k)}{\partial U_i(x_t)} \frac{\partial U_i(x_t)}{\partial \theta_j} \\ &= \frac{1}{q(x_t|\Theta_k)} \left[ 1 - \sum_{l=1}^k (p(l|x_t) - \delta_{jl}) \ln U_l(x_t) \right] \alpha_j \frac{\partial q(x_t|\theta_j)}{\partial \theta_j}, \end{aligned} \quad (16)$$

where  $\delta_{ij}$  is the Kronecker function.

Letting

$$\lambda_i(t) = 1 - \sum_{l=1}^k (p(l|x_t) - \delta_{il}) \ln U_l(x_t), \quad i = 1, \dots, k \quad (17)$$

and according to Eqs. (15) and (16), we have the following general adaptive gradient rules at sample  $x_t$ :

$$\nabla_{\beta_j} J_t(\Theta_k) = \frac{1}{q(x_t|\Theta_k)} \sum_{i=1}^k \lambda_i(t) (\delta_{ij} - \alpha_j) U_i(x_t), \quad (18)$$

$$\nabla_{\theta_j} J_t(\Theta_k) = \frac{\lambda_j(t) \alpha_j}{q(x_t|\Theta_k)} \frac{\partial q(x_t|\theta_j)}{\partial \theta_j}. \quad (19)$$

As each  $q(x_t|\theta_j)$  takes the form of univariate or multivariate Poisson distribution, we can get the adaptive gradient rules of  $J(\Theta_k)$  for the mixture of univariate or multivariate Poisson distributions according to the above general adaptive gradient rules. That is, we need only to replace these  $\partial q(x_t|\theta_j)/\partial \theta_j$ , i.e., the derivative of  $q(x_t|\theta_j)$  with respect to  $\theta_j$ , by their particular expressions. Now, we give them for the multivariate Poisson distributions in the two cases as follows.

(1) *The correlation case (defined by Eq. (5))*: Letting  $x_t = (x_{t1}, x_{t2}, \dots, x_{tm})^T$  and  $\theta_j = (\theta_{j0}, \theta_{j1}, \dots, \theta_{jm})^T$ , we then have

$$\frac{\partial q(x_t|\theta_j)}{\partial \theta_j} = \left( \frac{\partial q(x_t|\theta_j)}{\partial \theta_{j0}}, \frac{\partial q(x_t|\theta_j)}{\partial \theta_{j1}}, \dots, \frac{\partial q(x_t|\theta_j)}{\partial \theta_{jm}} \right)^T. \quad (20)$$

If there are some components of  $x_t$  which are zero, i.e., there is a nonempty subset  $H$  of  $\{1, 2, \dots, m\}$  such that  $x_{th} = 0$  iff  $h \in H$ , the derivatives of  $q(x_t|\theta_j)$  with respect to these  $\theta_{jh}$  (i.e.,  $h \in H$ ) take the following simple form (noting that  $0! = 1$ ):

$$\frac{\partial q(x_t|\theta_j)}{\partial \theta_{j0}} = \frac{\partial q(x_t|\theta_j)}{\partial \theta_{jh}} = -\exp \left( -\sum_{i=0}^m \theta_{ji} \right) \prod_{i=1}^m \frac{\theta_{ji}^{x_{ti}}}{x_{ti}!}. \quad (21)$$

As for the derivatives of  $q(x_t|\theta_j)$  with respect to  $\theta_{jh'}$  where  $h' \notin H$ , we have

$$\frac{\partial q(x_t|\theta_j)}{\partial \theta_{jh'}} = \exp \left( -\sum_{i=0}^m \theta_{ji} \right) \left( \frac{x_{th'}}{\theta_{jh'}} - 1 \right) \prod_{i=1}^m \frac{\theta_{ji}^{x_{ti}}}{x_{ti}!}. \quad (22)$$

On the other hand, if there is no zero component in  $x_t$ , we have the derivatives of  $q(x_t|\theta_j)$  with respect to each  $\theta_{jh}$  as follows:

$$\frac{\partial q(x_t|\theta_j)}{\partial \theta_{jh}} = q(x_t|\theta_j) \left( \frac{x_{th}}{\theta_{jh}} - 1 \right) - V_n(\theta_j), \quad (h > 0) \quad (23)$$

$$\frac{\partial q(x_t|\theta_j)}{\partial \theta_{j0}} = V_0(\theta_j) - q(x_t|\theta_j), \quad (24)$$

where

$$V_n(\theta_j) = \exp \left( - \sum_{i=0}^m \theta_{ji} \right) \prod_{i=1}^m \frac{\theta_{ji}^{x_{ti}}}{x_{ti}!} \sum_{i=1}^s \prod_{l=1}^m \binom{x_{tl}}{i} i! \left( \frac{\theta_{j0}}{\prod_{k=1}^m \theta_{jk}} \right)^i \frac{i}{\theta_{jn}}$$

for  $n = 0, 1, \dots, m$ .

(2) *The independent case (defined by Eq. (6))*: In this situation, the sample  $x_t$  is represented in the same way, but there is no component  $\theta_{j0}$  in  $\theta_j$ . When some component  $x_{th}$  of  $x_t$  is zero, the derivative of  $q(x_t|\theta_j)$  with respect to the corresponding  $\theta_{jh}$  takes the following simple form:

$$\frac{\partial q(x_t|\theta_j)}{\partial \theta_{jh}} = - \prod_{i=1}^m \frac{\theta_{ji}^{x_{ti}}}{x_{ti}!} e^{-\theta_{ji}}. \quad (25)$$

Otherwise, for a general component  $x_{th}$  of  $x_t$  that is not zero, the derivative with respect to the corresponding  $\theta_{jh}$  takes the following slightly complicated expression:

$$\frac{\partial q(x_t|\theta_j)}{\partial \theta_{jh'}} = \left( \frac{x_{th'}}{\theta_{jh'}} - 1 \right) \prod_{i=1}^m \frac{\theta_{ji}^{x_{ti}}}{x_{ti}!} e^{-\theta_{ji}}. \quad (26)$$

For the situation where each component is expressed by Eq. (1), i.e., a univariate Poisson distribution, it is certainly a special independent case of Eq. (6).

Summing up all these derivations, we finally obtain the adaptive gradient BYY learning rule for Poisson mixture as follows:

$$\beta_j^{new} = \beta_j^{old} + \eta \nabla_{\beta_j} J_t(\Theta_k), \quad (27)$$

$$\theta_j^{new} = \theta_j^{old} + \eta \nabla_{\theta_j} J_t(\Theta_k), \quad (28)$$

where  $\eta (> 0)$  denotes the learning rate that starts from a reasonable initial value and then reduces to zero with the iteration number  $n$  in such a way that  $0 \leq \eta(n) \leq 1$ , and

$$\lim_{n \rightarrow \infty} \eta(n) = 0, \quad \sum_{n=1}^{\infty} \eta(n) = \infty. \quad (29)$$

The typical example of the learning rate satisfying Eq. (29) is  $\eta(n) = \eta_0/n$ , where  $\eta_0$  is a positive constant.

#### 4. Simulation results and comparisons

In this section, simulation experiments are carried out to demonstrate the performance of the adaptive gradient BYY learning algorithm for Poisson mixture for both model selection and parameter estimation on a sample data set from a Poisson mixture, being compared with that of the unsupervised learning algorithm [31] for Poisson mixture.

##### 4.1. Sample data sets

To test our proposed adaptive gradient BYY learning algorithm for Poisson mixture, we generate eight typical sample data sets  $\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_8$  from finite mixtures of Poisson distributions of different types (e.g. univariate, bivariate, trivariate, correlation and independent Poisson distributions) with different sample sizes or mixing proportions, and the parameters of these Poisson mixtures are summarized in Table 1.

**Table 1**

The parameters of the eight (original) Poisson mixtures to generate sample data sets for simulation experiments.

Data set	Poissons	$\theta_1^i$	$\theta_2^i$	$\theta_0^i/\theta_3^i$	$\alpha_i$	$N_i$
$\mathcal{S}_1(k^* = 1)$	Poisson 1	6.0			1.0	300
$\mathcal{S}_2(k^* = 2)$	Poisson 1	1.0			0.20	100
	Poisson 2	10.0			0.80	400
$\mathcal{S}_3(k^* = 2)$	Poisson 1	1.0	2.0		0.20	100
	Poisson 2	7.0	8.0		0.80	400
$\mathcal{S}_4(k^* = 2)$	Poisson 1	3.0	2.0		0.50	250
	Poisson 2	12.0	14.0		0.50	250
$\mathcal{S}_5(k^* = 2)$	Poisson 1	1.0	2.0	1.0	0.40	200
	Poisson 2	11.0	12.0	8.0	0.60	300
$\mathcal{S}_6(k^* = 3)$	Poisson 1	1.0	2.0		1/3	200
	Poisson 2	10.0	11.0		1/3	200
	Poisson 3	23.0	22.0		1/3	200
$\mathcal{S}_7(k^* = 3)$	Poisson 1	3.0	2.0		1/3	200
	Poisson 2	9.0	10.0		1/3	200
	Poisson 3	15.0	16.0		1/3	200
$\mathcal{S}_8(k^* = 3)$	Poisson 1	1.0	2.0	1.0	1/3	200
	Poisson 2	4.0	5.0	6.0	1/3	200
	Poisson 3	10.0	12.0	11.0	1/3	200

Specifically, the first sample data set  $\mathcal{S}_1$  contains the sample data only from one univariate Poisson distribution, just as a degenerated Poisson mixture, while the other sample data sets contain the sample data from two or three Poisson distributions which always have some overlap, but keep separated in certain sense. The generating Poisson variables are univariate, bivariate and trivariate for  $\mathcal{S}_1$  to  $\mathcal{S}_2, \mathcal{S}_3$  to  $\mathcal{S}_7$  and  $\mathcal{S}_8$ , respectively. Moreover, the sample data in  $\mathcal{S}_5$  are generated from a correlation Poisson mixture, while the sample data in any of the other sample data sets are generated from an independent Poisson mixture. Particularly for  $\mathcal{S}_5$ , the two component Poisson variables are correlated via a common Poisson variable with parameter  $\theta_0$  as described previously in Section 2.2. For illustration, we sketch the sample data or points in each of  $\mathcal{S}_3, \dots, \mathcal{S}_8$ , respectively, in Fig. 1, from which we can observe that the component Poisson distributions in  $\mathcal{S}_3, \dots, \mathcal{S}_6$  are strongly separated, while those in  $\mathcal{S}_7$  and  $\mathcal{S}_8$  are overlapped in a higher degree.

##### 4.2. Simulation results

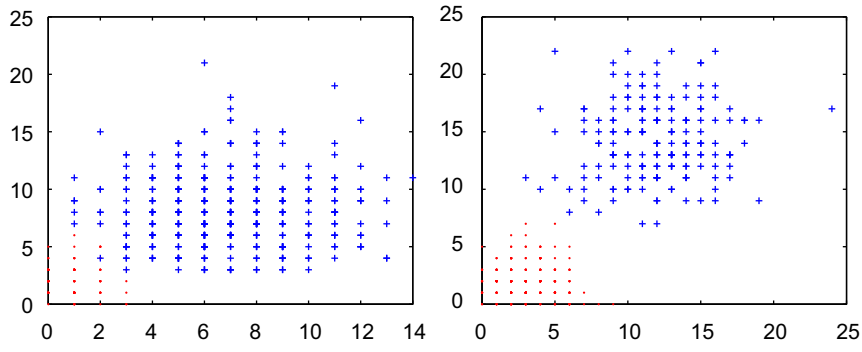
We implement the adaptive gradient BYY learning algorithm for Poisson mixture on these eight synthetic data sets with  $k \geq k^*$ . The parameters of the adaptive gradient BYY learning algorithm are initialized randomly in some intervals under the constraints. Particularly in our experiments, we initialize the parameters as follows. At first, we randomly set the initial values of the mixing proportions  $\alpha_j$  under the constraints that  $\alpha_j \geq 0$  and  $\sum_{j=1}^k \alpha_j = 1$ . Then, according to these given mixing proportions, we divide all the samples into  $k$  classes. In the independent case of Poisson mixture, each component parameter vector  $\theta_j$  is initialized as the corresponding sample mean vector of the  $j$ -th class. In the correlation case for  $\mathcal{S}_5$ , according to Eqs. (3) and (4), we have the following probability relations on each Poisson component  $j$ :

$$E(X_j) = (\theta_{j1} + \theta_{j0}, \theta_{j2} + \theta_{j0})^T, \quad (30)$$

$$\text{Var}(X_j) = \begin{pmatrix} \theta_{j1} + \theta_{j0} & \theta_{j0} \\ \theta_{j0} & \theta_{j2} + \theta_{j0} \end{pmatrix}. \quad (31)$$

When we use the sample mean vector and covariance matrix of  $\mathcal{S}_5$  instead of the mean vector in Eq. (30) and the covariance matrix in Eq. (31), respectively, we can get the two equations from which the





**Table 3**

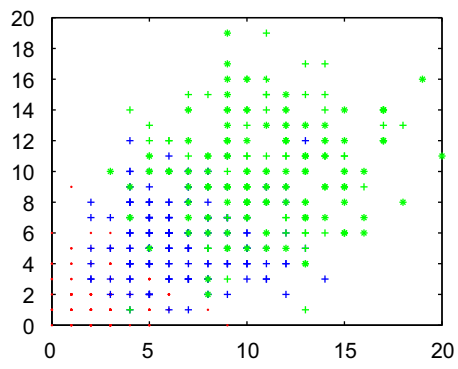
The comparisons of the AGL-BYY and UL-MML algorithms on the average implementation times.

Data set	Initial value of $k$	UL-MML (s)	AGL-BYY (s)
$\mathcal{S}_2(N=500)$	$2k^* - 1$	0.8988	1.9341
	$2k^*$	2.3256	1.7743
	$2k^* + 1$	4.0159	2.2281
$\mathcal{S}_3(N=500)$	$2k^* - 1$	1.3563	1.9877
	$2k^*$	3.0686	1.7986
	$2k^* + 1$	6.13325	1.9381
$\mathcal{S}_4(N=500)$	$2k^* - 1$	1.3969	1.8689
	$2k^*$	0.8672	1.8789
	$2k^* + 1$	3.7414	2.4684
$\mathcal{S}_6(N=600)$	$2k^* - 1$	2.9780	3.8177
	$2k^*$	4.7775	5.2666
	$2k^* + 1$	28.926	5.6242
$\mathcal{S}_7(N=600)$	$2k^* - 1$	4.0789	6.7984
	$2k^*$	23.8398	7.3975
	$2k^* + 1$	29.7119	8.3292

mixture is referred to as the AGL-BYY algorithm, while the unsupervised learning algorithm for Poisson mixture based on the MML criterion is referred to as the UL-MML algorithm. For simplicity, we only consider the sets of sample data from the independent Poisson mixtures. For both algorithms, the threshold value for the annihilation mechanism is set as 0.01. On each sample data set, we implement the two algorithms for 100 times with different initial settings and compare their average implementation times and the average parameter estimation accuracies. The AGL-BYY algorithm is implemented with the initial setting as above in the previous subsection. In this situation, the AGL-BYY algorithm can still always lead to the correct model selection. However, if the initial mixing proportions are randomly set as above, the UL-MML algorithm often leads to a wrong result on model selection. In order to overcome this weakness, we set each initial mixing proportion by  $1/k$ , i.e.,  $\alpha_j = 1/k$ . (Here  $k$  is severed as  $k_{max}$  in the unsupervised learning algorithm on finite mixture [31].) Accordingly, the sample data are equally divided into  $k$  classes and each component parameter vector  $\theta_j$  is also initialized as the corresponding sample mean vector of the  $j$ -th class. With such a parameter initial setting, the UL-MML algorithm generally leads to a correct model selection. Certainly, by different divisions of the sample data, we can get different parameter initial settings. For comparison, we only record the experimental results of the UL-MML algorithm that are successful on model selection.

We first compare the AGL-BYY and UL-MML algorithms on implementation time or convergence speed. The average implementation times (seconds) of the AGL-BYY and UL-MML algorithms on the typical sample data sets  $\mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4, \mathcal{S}_6$  and  $\mathcal{S}_7$  for  $k =$





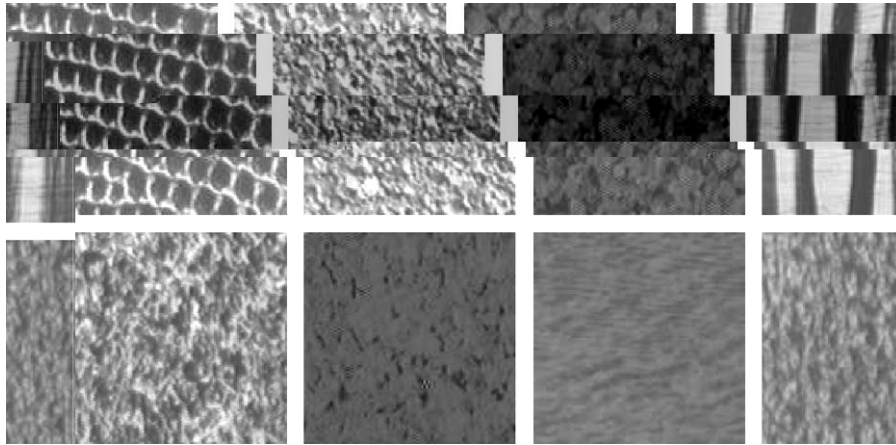


Fig. 3. The gray images: D03, D04, D29, D51, D09, D32, D38 and D24.

location and shape by learning these two kinds of independent parameters.

### 5. Application to texture classification

In this section, for practical usage and test, we apply the adaptive gradient BYY learning algorithm for Poisson mixture to texture classification or recognition on gray-scale Brodatz textures,<sup>1</sup> and compare it with two current state-of-the-art approaches.

#### 5.1. Poisson mixture learning based texture classification

As is well known, texture is a basic characteristic of an image and texture classification is a fundamental problem in image analysis and computer vision [43]. Actually, the task of texture classification (or recognition) is to classify images into a number of classes with different textures and the major difficulty relies on the description and measure of texture on an image. In the literature, the main focus is on the extraction techniques of texture features. In this work, we use the Poisson mixture model to directly describe the gray images with a particular texture so that our adaptive gradient BYY learning algorithm can be applied to texture classification on gray images.

We adopt the spatial analysis technique for gray images which is fundamental for texture classification [44]. Via this technique, a discrete gray image can be decomposed as a series of gray-level planes defined as follows:

$$f_g(u, v) = \begin{cases} 1, & f(u, v) = g, \\ 0, & f(u, v) \neq g, \end{cases} \quad (32)$$

where  $(u, v)$  denotes a pair of discrete coordinates for a pixel at the image,  $f(u, v)$  is the discrete gray level of the image at the pixel  $(u, v)$ , and  $f_g(u, v)$  is just the gray plane with gray-level  $g$ . In this way, the stochastic nature of a gray image can be described through the distributions of the numbers of different gray-level points within a patch, i.e., a square region or block, picked up in the image. Supposing that  $\{0, 1, \dots, G - 1\}$  is the set of gray levels in consideration, as we pick up a patch  $B$  of  $L \times L$  pixels in the image, we get a random feature vector  $\mathbf{W} = (W_0, \dots, W_{G-1})^T$ , where each component  $W_g$  is just the number of points in  $B$  with gray-level  $g$ , being computed by

$$W_g = \sum_{u, v \in B} f_g(u, v). \quad (33)$$

With this random feature vector  $\mathbf{W}$ , we can construct a Bayesian classifier for texture classification. Firstly, we pick up a number of patches of a fixed size (i.e.,  $L \times L$  pixels) from the given gray images with different texture classes, being denoted by  $\{1, 2, \dots, C\}$ . Then, we can get the feature vectors  $\mathbf{W}$ s for those image patches with the help of Eq. (33) as well as their corresponding texture indexes. Based on these sample data, we can train the Bayesian classifier through the posteriori probability given as follows:

$$P(T_c | \mathbf{W}) = \frac{P(\mathbf{W} | T_c)P(T_c)}{\sum_{i=1}^C P(\mathbf{W} | T_i)P(T_i)}, \quad (34)$$

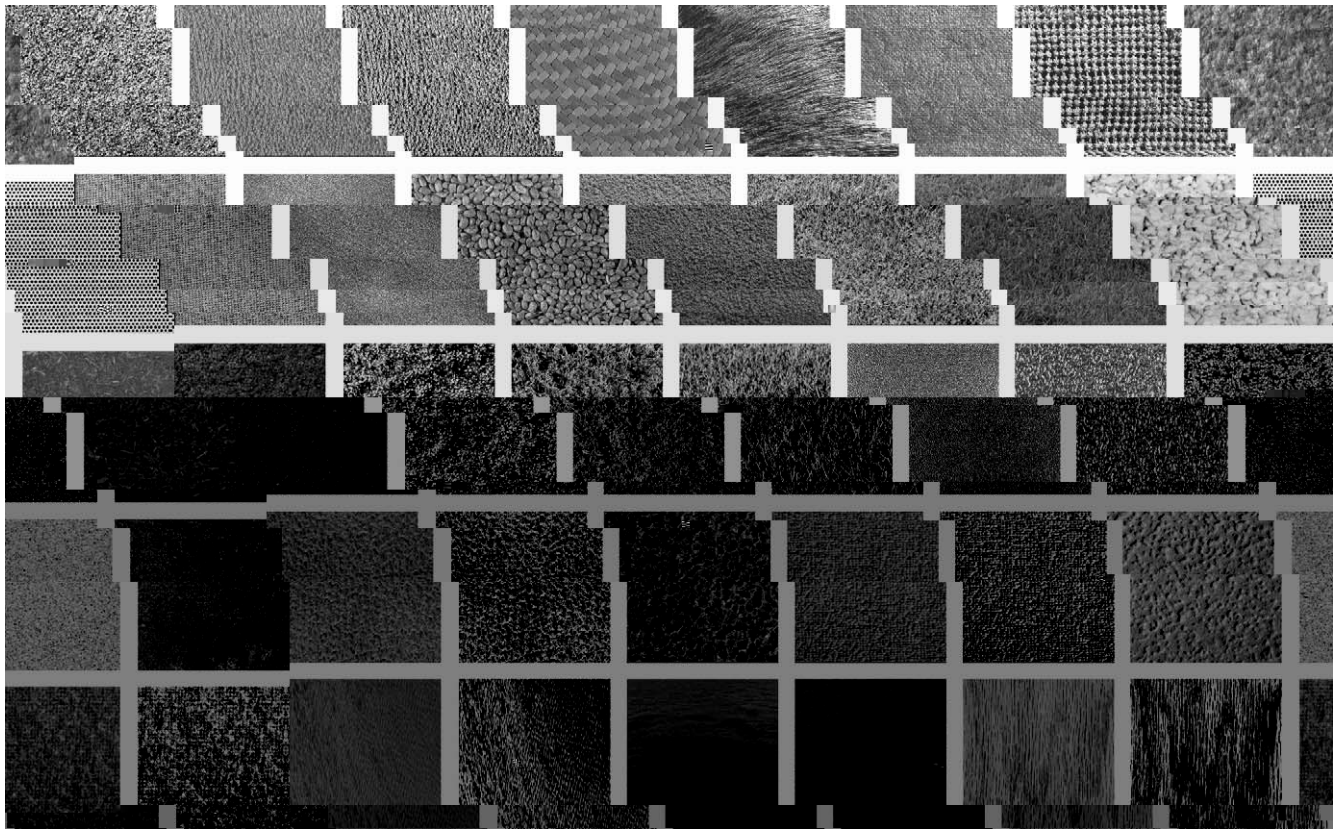
where  $T_c$  represents texture class  $c$ ,  $P(T_c)$ ,

<sup>1</sup> Brodatz texture images from <http://www.cipr.rpi.edu/resource/stills/brodatz.html>.

**Table 5**  
The results of the texture classification on the eight gray images.

DC	RC1	RC2	RC3	RC4	RC5	RC6	RC7	RC8	CAR (%)
DC1	254	2	0	0	0	0	0	0	99.22
DC2	1	249	0	0	6	0	0	0	97.27
DC3	0	0	256	0	0	0	0	0	100
DC4	5	5	1	237	6	0	0	2	92.58
DC5	0	22	0	0	234	0	0	0	91.41
DC6	0	0	0	0	0	256	0	0	100
DC7	0	0	0	0	0	0	256	0	100
DC8	0	1	0	0	10	0	0	245	95.70

Here DC<sub>*i*</sub> represents data class *i*, RC<sub>*i*</sub> represents resulted class *i* and CAR represents classification accuracy rate.



**Fig. 4.** The 40 gray images used in [45,46].

5.2. Experimental results and comparisons

To test the effectiveness of the adaptive gradient BYY learning algorithm on the Poisson mixture based texture classification, we apply it to learning each  $P(W_i|\Theta_{i,c})$  for eight typical gray texture images (denoted by D03, D04, D29, D51, D09, D32, D38 and D24) with  $128 \times 128$  pixels and 0-255 gray levels from the Brodatz image base, which are, respectively, shown in Fig. 3. So, the number of texture classes is eight, i.e.,  $C = 8$ .

We begin to randomly pick up 128 image patches of  $16 \times 16$  pixels from each texture image, and compute their feature vectors to form a training data set. On the other hand, we randomly pick up the other 256 image patches of  $16 \times 16$  pixels from each texture image and compute their feature vectors to form a testing data set. During our training, if some component  $W_i$  is always 0 for every texture class, it is obvious that these  $P(W_i|\Theta_{i,c})$  have no influence on the texture classification and we need not consider these degenerated but unchanged

**Table 6**

The comparison results of the BYY learning method and the spectral histograms method.

Size of patches	Average classification accuracy rates of the two methods	
	Spectral histogram method (%)	BYY learning method (%)
24 × 24	89.65	90.48
32 × 32	92.50	94.65

texture classification on the testing data set from a general training experiment are listed in Table 5.

From those results, we can find that the total accuracy rate of texture classification on the whole eight texture classes is 97.2%, but the accuracy rates of texture classification on the first, second, third, sixth and seventh classes are quite high and three of them even reach at 100%, that is, all the testing samples in these three classes are classified correctly. Though the texture classification accuracy rates on the other three classes are relatively low (92.58%, 91.42% and 95.70%, respectively), they are still acceptable because there really exists certain similarity between the textures of some two images (like d04 and d09), especially in certain local regions.

For comparison, we select two current state-of-the-art approaches on texture classification. One is the spectral histogram method [45] which employs the spectral histogram of sub-band images obtained from a bank of given filters on an image window as the feature statistic for texture classification. The other is the bit-plane probability (BP) signature method [46] which models the wavelet sub-band histograms of an image window as the product Bernoulli distributions (PBD) and uses the bit-plane probabilities or parameters in these PBD models, i.e., the BP signatures, as the feature statistic for texture classification. For convenience, we consider the set of 40 gray 256 × 256 images from the Brodatz image base used in both [45,46], which are, respectively, shown in Fig. 4. Actually, this gray image or texture set is challenging because there are significant variations within some texture and some of them are very similar to each other.

In comparison with the spectral histogram method, we randomly pick up two disjoint sets of 256 image patches from each texture image to form a training and test data sets, respectively. As suggested in [45], we test on two sizes of patches. One size is 24 × 24 pixels, while the other is 32 × 32 pixels. As there are 40 gray images,  $G=256$ . The average classification accuracy rates of our BYY learning method on the two sizes of patches over 100 trials are listed in Table 6, being with those of the spectral histogram method obtained in [45]. It can be found from Table 6 that our BYY learning method is slightly better than the spectral histogram method on texture classification.

In order to compare our BYY learning method with the BP signature method on these 40 gray images, we conduct the above BYY learning experiments with the image patches of 48 × 48 pixels and find out that the average classification accuracy rate reaches at 97.59%. According to the fact that the classification accuracy rate of a texture classification method increases with the size of image patches, the average classification accuracy rate of our BYY learning method on the image patches of 128 × 128 pixels should be larger than 97.59%, which is considerably higher than 96.8%, the average classification accuracy rate of the BP signature method for these 40 gray images on the image patches of 128 × 128 pixels obtained in [46]. Thus, we can believe that our BYY learning method is more efficient than the BP signature method on texture classification.

In summary, our adaptive gradient BYY learning algorithm is able to automatically determine the number of actual Poisson components in the real-world data from a gray image and construct a good Poisson mixture model for them, which can be successfully applied

to the texture classification and even better than the two current state-of-the-art texture classification methods.

## 6. Conclusions

We have applied the Bayesian Ying–Yang harmony learning mechanism to the Poisson mixture modeling for parameter learning with automated model selection by constructing an adaptive gradient BYY learning algorithm for Poisson mixture. It is demonstrated by the simulation experiments that with a sample data set, our proposed adaptive gradient learning algorithm not only determines the number of actual Poisson components automatically during parameter learning, but also obtains a good estimation of the parameters in the original Poisson mixture, as long as those actual Poisson components are separated in a certain degree. Moreover, it outperforms the unsupervised learning algorithm for Poisson mixture on model selection. Moreover, the adaptive gradient learning algorithm can be used to build an efficient Bayesian classifier for texture classification on gray images.

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