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Abstract.

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g h f s f 6 h 6 r f
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sh - hss s br 6 s.

I

$y \quad Wx \quad W \text{ As} \quad WA \text{ s}$

$$\mathbf{y} \in \mathbb{R}^n \text{, } s \in S \text{, } \mathbf{x} \in \mathcal{M} \text{, } \mathbf{y} \in \mathcal{S} \text{, } s \in S \text{, } \mathbf{x} \in \mathcal{M} \text{, } \mathbf{y} \in \mathcal{S} \text{, } s \in S \text{, } \mathbf{x} \in \mathcal{M} \text{, } \mathbf{y} \in \mathcal{S} \text{, } s \in S \text{, } \mathbf{x} \in \mathcal{M} \text{, } \mathbf{y} \in \mathcal{S}$$

$$q \text{ } \mathbf{y} \quad \prod_{i=1}^n q \text{ } y_i ,$$

$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} \mathbf{C}_1 & \mathbf{C}_2 \\ \mathbf{C}_3 & \mathbf{C}_4 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{D}_3 & \mathbf{D}_4 \end{pmatrix}$, $\mathbf{E} = \begin{pmatrix} \mathbf{E}_1 & \mathbf{E}_2 \\ \mathbf{E}_3 & \mathbf{E}_4 \end{pmatrix}$, $\mathbf{F} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 \\ \mathbf{F}_3 & \mathbf{F}_4 \end{pmatrix}$, $\mathbf{G} = \begin{pmatrix} \mathbf{G}_1 & \mathbf{G}_2 \\ \mathbf{G}_3 & \mathbf{G}_4 \end{pmatrix}$, $\mathbf{H} = \begin{pmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ \mathbf{H}_3 & \mathbf{H}_4 \end{pmatrix}$, $\mathbf{I} = \begin{pmatrix} \mathbf{I}_1 & \mathbf{I}_2 \\ \mathbf{I}_3 & \mathbf{I}_4 \end{pmatrix}$, $\mathbf{J} = \begin{pmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{pmatrix}$, $\mathbf{K} = \begin{pmatrix} \mathbf{K}_1 & \mathbf{K}_2 \\ \mathbf{K}_3 & \mathbf{K}_4 \end{pmatrix}$, $\mathbf{L} = \begin{pmatrix} \mathbf{L}_1 & \mathbf{L}_2 \\ \mathbf{L}_3 & \mathbf{L}_4 \end{pmatrix}$, $\mathbf{M} = \begin{pmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_3 & \mathbf{M}_4 \end{pmatrix}$, $\mathbf{N} = \begin{pmatrix} \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{N}_3 & \mathbf{N}_4 \end{pmatrix}$, $\mathbf{O} = \begin{pmatrix} \mathbf{O}_1 & \mathbf{O}_2 \\ \mathbf{O}_3 & \mathbf{O}_4 \end{pmatrix}$, $\mathbf{P} = \begin{pmatrix} \mathbf{P}_1 & \mathbf{P}_2 \\ \mathbf{P}_3 & \mathbf{P}_4 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 \\ \mathbf{Q}_3 & \mathbf{Q}_4 \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} \mathbf{R}_1 & \mathbf{R}_2 \\ \mathbf{R}_3 & \mathbf{R}_4 \end{pmatrix}$, $\mathbf{S} = \begin{pmatrix} \mathbf{S}_1 & \mathbf{S}_2 \\ \mathbf{S}_3 & \mathbf{S}_4 \end{pmatrix}$, $\mathbf{T} = \begin{pmatrix} \mathbf{T}_1 & \mathbf{T}_2 \\ \mathbf{T}_3 & \mathbf{T}_4 \end{pmatrix}$, $\mathbf{U} = \begin{pmatrix} \mathbf{U}_1 & \mathbf{U}_2 \\ \mathbf{U}_3 & \mathbf{U}_4 \end{pmatrix}$, $\mathbf{V} = \begin{pmatrix} \mathbf{V}_1 & \mathbf{V}_2 \\ \mathbf{V}_3 & \mathbf{V}_4 \end{pmatrix}$, $\mathbf{W} = \begin{pmatrix} \mathbf{W}_1 & \mathbf{W}_2 \\ \mathbf{W}_3 & \mathbf{W}_4 \end{pmatrix}$, $\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 \\ \mathbf{X}_3 & \mathbf{X}_4 \end{pmatrix}$, $\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_1 & \mathbf{Y}_2 \\ \mathbf{Y}_3 & \mathbf{Y}_4 \end{pmatrix}$, $\mathbf{Z} = \begin{pmatrix} \mathbf{Z}_1 & \mathbf{Z}_2 \\ \mathbf{Z}_3 & \mathbf{Z}_4 \end{pmatrix}$.

$$D(\mathbf{W}) = -H(\mathbf{y}) - \sum_{i=1}^n \int p_{\mathbf{W}}(y_i | \mathbf{W}) \log p_i(y_i) dy_i,$$

m a s s s s n I A m
 globally m x m a s m
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 s m s a s s n I A m
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$$J \in \mathbf{R} \quad \sum_{i=1}^n \sum_{j=1}^n r_{ij}^4 \nu_j^s k_i^m,$$

$$\mathbf{K} = k_{ij} \in \mathbb{R}^{n \times n}, \quad k_{ij} = \nu_j^s k_i^m.$$

$$n \quad m \quad , \quad , \quad s$$

$$J \in \mathbf{R} \quad \sum_{i=1}^n \sum_{j=1}^n r_{ij}^4 \nu_j^s k_i^m \quad \sum_{i=1}^n \sum_{j=1}^n r_{ij}^4 k_{ij}.$$

$$k_1^m \geq \cdots \geq k_p^m > k_{p+1}^m \geq \cdots \geq k_n^m, \quad \nu_1^s \geq \cdots \geq \nu_p^s > \nu_{p+1}^s \geq \cdots \geq \nu_n^s$$

Lemma 1. Suppose that $F \times (\mathbf{x} \in \mathbb{R}^m)$ is a twice differentiable scalar function under the following constraints:

$$C_i \mathbf{x} \quad , \quad i = 1, \dots, k.$$

Construct a Lagrange function with a Lagrange multiplier set $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_k\}$, i.e., $L(\mathbf{x}, \lambda) = F(\mathbf{x}) - \sum_{i=1}^k \lambda_i C_i(\mathbf{x})$, and assume that \mathbf{x}^*, λ^* is a solution of the system of the equalities that all the derivatives of $L(\mathbf{x}, \lambda)$ with respect to the variables of \mathbf{x} and the Lagrange multipliers λ_i are equal to zeros. It is also assumed that these $\nabla C_i(\mathbf{x}^*)$ are linearly independent. If for any nonzero vector \mathbf{q} under the constraints $\mathbf{q}^T \nabla C_i(\mathbf{x}^*) = 0$ for $i = 1, \dots, k$, we have

$$\mathbf{q}^T \nabla^2 L(\mathbf{x}^*, \boldsymbol{\lambda}^*) \mathbf{q} < 0 \quad or \quad ,$$

then \mathbf{x}^* is a local maximum (or local minimum) of F \mathbf{x} under the constraints.

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$J \mathbf{R}$

$$R^T R = I_m \quad m \times m \quad m \times m \quad m \times m \quad m \times m \quad x \times s$$

$$R^T R = I_m \quad m \times m \quad m \times m \quad m \times m \quad x \times m \quad J \quad R \quad m \times x \quad x \times m \quad x \times K$$

$$I_m \quad m \times m \quad m \times m \quad m \times m \quad m \times m \quad s \times s$$

$$R^T R = I_m \quad m \times m \quad m \times m \quad m \times m \quad x \times s \quad \lambda \quad \{\lambda_{ij} \mid i \leq j\} \quad s \times s \quad n \times T \quad T \times s$$

$$L(\mathbf{R}, \boldsymbol{\lambda}) = \sum_{i=1}^n \sum_{j=1}^n r_{ij}^4 k_{ij} - \sum_{i=1}^n \sum_{j=i}^n \lambda_{ij} \left(\sum_{l=1}^n r_{li} r_{lj} - \delta_{ij} \right),$$

$$a_{ij} \quad \delta_{ij} \quad \dots \quad s \quad a \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad a$$

$$\frac{\partial L(\mathbf{R}, \boldsymbol{\lambda})}{\partial r} = 4k \cdot r^3 + \sum_{i=1}^n r_i \lambda_i + 2\lambda \cdot r + \sum_{i=n+1}^n r_i \lambda_i; \quad (10)$$

$$\frac{\partial L(\mathbf{R}, \boldsymbol{\lambda})}{\partial \lambda} = \sum_{i=1}^n r_i r_i - \delta \quad . \quad (11)$$

$$\lambda \quad \quad \quad m \quad \text{x} \quad \mathbf{U} \quad \quad u_{ij} \quad n \times n \quad s$$

$$u = \begin{cases} \lambda \end{cases}$$

Theorem 1. If \mathbf{R}^* is a permutation matrix up to sign indeterminacy and $k_{ij} > 0$ at all the positions where $|r_{ij}^*| = 0$, it corresponds to a local maximum of the objective function $J(\mathbf{R})$.

Proof:

$$\text{vec } \mathbf{R} = [r_{11}, r_{21}, \dots, r_{n1}, r_{12}, r_{22}, \dots, r_{n2}, \dots, r_{1n}, r_{2n}, \dots, r_{nn}]^T \in \mathbb{R}^{n^2}.$$

$$\mathbf{q} = [q_{11}, q_{21}, \dots, q_{n1}, q_{12}, q_{22}, \dots, q_{n2}, \dots, q_{1n}, q_{2n}, \dots, q_{nn}]^T \in \mathbb{R}^{n^2}.$$

$$\frac{\partial^2 L(\mathbf{R}, \lambda)}{\partial r_{ij} \partial r_{i'j'}} = \delta_{(i,j),(i',j')} - k_{ij} r_{ij}^2 - u_{jj},$$

$$\frac{\partial}{\partial r_{ij}} \left[\frac{\delta_{(i,j),(i',j')}}{k_{ij} r_{ij}^2 + u_{jj}} \right] = \frac{\delta_{(i,j),(i',j')}}{k_{ij} r_{ij}^2 + u_{jj}} - \frac{2k_{ij}}{(k_{ij} r_{ij}^2 + u_{jj})^2} \frac{\partial}{\partial r_{ij}} (k_{ij} r_{ij}^2 + u_{jj}) = \frac{\delta_{(i,j),(i',j')}}{k_{ij} r_{ij}^2 + u_{jj}} - \frac{2k_{ij}^2}{(k_{ij} r_{ij}^2 + u_{jj})^2} r_{ij}.$$

$$\mathbf{U} = \mathbf{R}^T \mathbf{B}.$$

$$\frac{\partial}{\partial r_{ij}} \left[\frac{\delta_{(i,j),(i',j')}}{k_{ij} r_{ij}^2 + u_{jj}} \right] = \frac{\delta_{(i,j),(i',j')}}{k_{ij} r_{ij}^2 + u_{jj}} - \frac{2k_{ij}^2}{(k_{ij} r_{ij}^2 + u_{jj})^2} r_{ij} < 0 \quad \text{everywhere}$$

$$\begin{aligned} & \text{Left side: } \sum_{i=1}^n \sum_{j=1}^m k_{ij} \leq \sum_{i=1}^n \sum_{j=1}^m s_i x_j \\ & \text{Right side: } \sum_{i=1}^n \sum_{j=1}^m s_i x_j = \sum_{i=1}^n s_i \sum_{j=1}^m x_j = \sum_{i=1}^n s_i m = \sum_{i=1}^n s_i \sum_{j=1}^m m_j = \sum_{i=1}^n s_i m_i \end{aligned}$$

Theorem 2. If \mathbf{R}^* is a permutation matrix up to sign indeterminacy and $k_{ij} < 0$ at all the positions where $|r_{ij}^*| = 1$, it corresponds to a local minimum of the objective function $J(\mathbf{R})$.

Remark 2. A

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 m m n m x s m m J R s n s .
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1. $\text{f} \cdot \text{A} \cdot \text{f} \cdot \text{A} \cdot \text{f} \cdot \text{A} \cdot \text{f} \cdot \text{A} \cdot \text{f} \cdot \text{A}$, 41(7)(1993) 2461-2470
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