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Abstract.

I

A $\mathbf{A} \in \mathbb{R}^{m\times n}$ $\mathbf{s} \in \mathbb{R}^n$ $\mathbf{x} \in \mathbb{R}^m$ n m $\mathbf{W} \in \mathbb{R}^{n\times m}$ $\mathbf{x}$

$$\mathbf{y} \quad \mathbf{Wx} \quad \mathbf{W} \; \mathbf{As} \quad \mathbf{WA} \; \mathbf{s}$$

$\mathbf{y} \in \mathbb{R}^n$ $\mathbf{s}$ $\mathbf{y}$

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$$q(\mathbf{y}) \;\; \prod_{i=1}^{n} q(y_i),$$

$$D(\mathbf{W}) \;\; -H(\mathbf{y}) - \sum_{i=1}^{n} \int p(y_i | \mathbf{W}) \; p_i(y_i)\, dy_i,$$

[illegible] *globally* [illegible] *locally* [illegible]

[illegible]

$$J(\mathbf{R}) = \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^{4}\nu_j^{s}k_i^{m},$$

[illegible] $\mathbf{R} = (r_{ij})_{n\times n}$ [illegible] **WA** [illegible] $\mathbf{R}$ [illegible] **W** [illegible] ν_j^{s} [illegible] s_j [illegible] k_i^{m} [illegible] ν_i^{m} [illegible] $\mathbf{K}$

$$\mathbf{K} = (k_{ij})_{n\times n}, \quad k_{ij} = \nu_j^{s}k_i^{m}.$$

$$J(\mathbf{R}) = \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^{4}\nu_j^{s}k_i^{m} - \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^{4}k_{ij}.$$

$k_1^m \geq \cdots \geq k_p^m > [illegible] > k_{p+1}^m \geq \cdots \geq k_n^m$ [illegible] $\nu_1^s \geq \cdots \geq \nu_p^s > [illegible] > \nu_{p+1}^s \geq \cdots \geq \nu_n^s$ [illegible]

Lemma 1. *Suppose that* $F(\mathbf{x})$ $(\mathbf{x} \in \mathbb{R}^m)$ *is a twice differentiable scalar function under the following constraints:*

$$C_i(\mathbf{x}) = [illegible], \quad i = [illegible], [illegible], \cdots, k.$$

Construct a Lagrange function with a Lagrange multiplier set $\boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \cdots, \lambda_k\}$, *i.e.,* $L(\mathbf{x}, \boldsymbol{\lambda}) = F(\mathbf{x}) - \sum_{i=1}^{k} \lambda_i C_i(\mathbf{x})$, *and assume that* $(\mathbf{x}^*, \boldsymbol{\lambda}^*)$ *is a solution of the system of the equalities that all the derivatives of* $L(\mathbf{x}, \boldsymbol{\lambda})$ *with respect to the variables of* $\mathbf{x}$ *and the Lagrange multipliers* λ_i *are equal to zeros. It is also assumed that these* $\nabla C_i(\mathbf{x}^*)$ *are linearly independent. If for any nonzero vector* [illegible] *under the constraints* $\mathbf{q}^T \nabla (C_i(\mathbf{x}^*)) = [illegible]$ *for* $i = [illegible], [illegible], \cdots, k$, *we have*

$$\mathbf{q}^T \nabla^2 L(\mathbf{x}^*, \boldsymbol{\lambda}^*)\mathbf{q} < [illegible] \quad (or > [illegible]),$$

then $\mathbf{x}^*$ *is a local maximum (or local minimum) of* $F(\mathbf{x})$ *under the constraints.*

s m m s s n m m x s n m m x m , J **R** n x m x **K** **RR**^{T} **I**,

s n s s m m s m s $\boldsymbol{\lambda}$ $\{\lambda_{ij}$ $i \leq j\}$ s n

$$L(\mathbf{R}, \boldsymbol{\lambda}) = \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij}^4 k_{ij} + \sum_{i=1}^{n} \sum_{j=i}^{n} \lambda_{ij} \Big(\sum_{l=1}^{n} r_{li} r_{lj} - \delta_{ij} \Big),$$

n δ_{ij} s n n

$$\frac{\partial L(\mathbf{R}, \boldsymbol{\lambda})}{\partial r} = 4k \; r^3 + \sum_{=1}^{1} r \; \lambda \; + 2\lambda \; r \; + \sum_{= +1}^{n} r \; \lambda \;; \qquad (10)$$

$$\frac{\partial L(\mathbf{R}, \boldsymbol{\lambda})}{\partial \lambda} = \sum_{=1}^{n} r \; r \; - \delta \; . \qquad (11)$$

$\boldsymbol{\lambda}$ m x **U** u_{ij} $n \times n$ s

$$u = \begin{cases} \lambda \\ \end{cases}$$

Theorem 1. *If* $\mathbf{R}^*$ *is a permutation matrix up to sign indeterminacy and* $k_{ij} >$ *at all the positions where* $|r^*_{ij}|$ *, it corresponds to a local maximum of the objective function* J $\mathbf{R}$ *.*

Proof: n $n \times n$ m x $\mathbf{R}$ $n^2 \times$

$$vec\,\mathbf{R} \quad r_{11}, r_{21}, \cdots, r_{n1}, r_{12}, r_{22}, \cdots, r_{n2}, \cdots, r_{1n}, r_{2n}, \cdots, r_{nn} \,{}^T \in \mathbb{R}^{n^2}.$$

s m s s $n^2 \times$ $\mathbf{q}$

$$\mathbf{q} \quad q_{11}, q_{21}, \cdots, q_{n1}, q_{12}, q_{22}, \cdots, q_{n2}, \cdots, q_{1n}, q_{2n}, \cdots, q_{nn} \,{}^T \in \mathbb{R}^{n^2}.$$

n s

$$\frac{\partial^2 L \;\; \mathbf{R}, \boldsymbol{\lambda}}{\partial r_{ij} \partial r_{i'j'}} \quad \delta_{(i,j),(i',j')} \quad k_{ij} r^2_{ij} \quad u_{jj} \,,$$

n $\delta_{(i,j),(i',j')}$ s n s n n s i', j' i, j m i i' j j' n s s m n

$$\mathbf{U} \quad - \; \mathbf{R}^T \mathbf{B}.$$

n $\mathbf{R}$ $\mathbf{R}^*$ s m m x s m $\mathbf{U}^*$ ss n $\boldsymbol{\lambda}^*$ m x n n $k_{ij} >$ *every* $|r^*_{ij}|$ s n u^*_{jj}

[illegible] $\mathbf{K}_{11}$ [illegible] $\mathbf{K}_{22}$ [illegible] k_{ij} [illegible] J $\mathbf{R}$ [illegible] $p!$ $n-p$ $!$ [illegible] $\le p \le n$ [illegible] J $\mathbf{R}$ [illegible] J $\mathbf{R}$ [illegible]

Theorem 2. *If* $\mathbf{R}^*$ *is a permutation matrix up to sign indeterminacy and* $k_{ij} <$ [illegible] *at all the positions where* $|r^*_{ij}|$ [illegible]*, it corresponds to a local minimum of the objective function* J $\mathbf{R}$ *.*

Remark 2. [illegible] $\mathbf{K}_{12}$ [illegible] $\mathbf{K}_{21}$ [illegible] J $\mathbf{R}$ [illegible] n [illegible] p [illegible]

[illegible] k_{ij} [illegible] $|r^*_{ij}|$ [illegible] $\mathbf{R}^*$ [illegible] J $\mathbf{R}$ [illegible]

[illegible] J $\mathbf{R}$ [illegible]

[illegible]

[illegible] J $\mathbf{R}$ [illegible]

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