A Fixed-Point EM Algorithm for Straight Line Detection

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Abstract. Straight line detection is a basic technique in image processing and pattern recognition. It has been investigated from different aspects, but is still very challenging in practical applications. In this paper, based on the finite mixture model and under the EM framework, we maximize the Q-function by differentiation and construct a fixed-point EM algorithm for straight line detection. It is demonstrated by the experiments that this proposed algorithm can effectively detect the straight lines from a digital image or dataset.

Keywords: Straight line detection, Expectation Maximization (EM), Fixed-Point iteration.

1 Introduction

Straight line detection is a basic technique in image processing and pattern recognition. In fact, it is a process of locating the straight lines from a digital image or 2-dimensional dataset and there are a variety of learning algorithms for straight line detection. In the literature, the Hough transform (HT) [1] and its extensions [2] are important tools for straight line detection. Generally, the Hough transform suffers from heavy computational cost. In order to alleviate this weakness, Randomized Hough Transform (RHT) [3] and the constrained Hough Transform [4] were further established. From the theoretical aspects, there have also been established many learning algorithms for straight line or curve detection (e.g., [5]-[8]).

The local principal component analysis (PCA) algorithm [9]-[12], as an extension of PCA [13], is often used for straight line detection. It implements the least mean square error reconstruction (LMSER) principle [14] and detects the straight lines via minimizing the following cost function [11]:

$$E = \sum_{k=1}^{K} E_k = \sum_{k=1}^{K} \sum_{x_t \in \mathcal{L}_k} d^2(x_t, \mathcal{L}_k) \tag{1}$$

where x_t is the t-th sample point belonging to the line $\mathcal{L}_k$ and $d(x_t, \mathcal{L}_k)$ denotes the Euclidean distance from the data point x_t to the line $\mathcal{L}_k$. Actually, the line $\mathcal{L}_k$ can be considered as a special set of points. On the other hand, the line $\mathcal{L}_k$

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ca be a egaded a a d e . Ac a , E eache i i i he he k- h aigh i e he fi i ci a c e ec f he d e $\mathcal{L}_k$. The ef e, he i f $\mathcal{L}_k$ j he a e a ha f he ca i ci a c e a a e h d.

Each x_t a ig ed a aigh i e b he c a ifica i e be hi f c i give b

$$I(x_t,k)=\begin{cases}1 & \text{if } k = \text{a g i } d(x_t,\mathcal{L}_j),\ j=1,\cdots,K\\ 0 & \text{he e.}\end{cases} \qquad (2)$$

I fac , x_t be g $\mathcal{L}_k$ if a d if $I(x_t,k)=1$. I ea ha he d a ce f x_t $\mathcal{L}_k$ he a e . Th , e ca da e $\mathcal{L}_k$ b he f i g e: $\mathcal{L}_k$ he fi i ci a c e f he e $\mathcal{L}_k$. Rece , he RPCL a g i h [15] a c bi ed i h he ca PCA a g i h f aigh i e de ec i [6].

I hi a e , e i i e he fi i e i e de f aigh i e de ec i . I a e e he a i i f he g i d f c i di ec . S , e e he E ec a i Ma i i a i (EM) [16] a d a a e he Q-f c i . B he diffe e ia i f he Q-f c i , e c c a fi ed- i EM a g i h f aigh i e de ec i . I de a ed b he e e i e ha he ed fi ed- i EM a g i h ca de ec he aigh i e f a da a e effec ive .

The e f he a e ga i ed a f . We begi i d ce he fi i e i e de f aigh i e de ec i i Sec i 2. We he de ive a d e fi ed- i EM a g i h i Sec i 3. The e e i e a e a e f he de a ed i Sec i 4. Fi a , a b ief c c i i ade i Sec i 5.

2 The Finite Mixture Model for Straight Line Detection

He e, i a ed ha he be f aigh i e i ea i g de e a he be f ac a aigh i e i he i age da a e . F i ici , e f c he 2-di e i a i a i , b he de iva i a d a a ca be ea i ge e a i ed he i a i i h highe di e i . Le he da a e be de ed b $\mathcal{S}=\{x_t\}^N$

where $(x_t - m_k, \ell_k)$ denotes the inner product of $x_t - m_k$ and ℓ_k.

In this situation, we can establish the following finite mixture model:

$$q(x|\Theta_K) = \sum_{j=1}^{K} \pi_j q(x|\ell_j, m_j, \sigma_j), \tag{7}$$

where

$$q(x|\ell_j, m_j, \sigma_j) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\{-\frac{d^2(x_t, \ell_j, m_j)}{2\sigma_j^2}\}, \tag{8}$$

$$\|\ell_j\|^2 = {\ell_{j1}}^2 + {\ell_{j2}}^2 = 1, \quad j = 1, \ldots, K, \tag{9}$$

$$\sum_{j=1}^{K} \pi_j = 1. \tag{10}$$

In this mixture model, π_i represents the mixing proportion. $d^2(x_t, \ell_k, m_k)$ denotes the distance between the point x_t and the line with parameters ℓ_k and m_k. σ_i can be considered as the noise level of the dataset. That is, when σ_i is large, the noise level is high.

As it is a finite mixture model, we are difficult to find the maximum of its log-likelihood function directly. So, we resort to the EM algorithm [16]. Under the EM framework, we introduce a missing variable j and construct the Q-function:

$$Q(\Theta_K^h, \Theta_K^{h+1}) = \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{K} p(j|x_t, \Theta_K^h) \ln q(x_t|j, \Theta_K^{h+1}) \tag{11}$$

$$= \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{K} \frac{\pi_j^h q(x_t|\ell_j^h, m_j^h, \sigma_j^h)}{\sum_{i=1}^{K} \pi_i^h q(x_t|\ell_j^h, m_j^h, \sigma_j^h)} \ln[\pi_j^{h+1} q(x_t|\ell_j^{h+1}, m_j^{h+1}, \sigma_j^{h+1})]$$

$$= \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{K} p_j(t) \ln[\pi_j^{h+1} q(x_t|\ell_j^{h+1}, m_j^{h+1}, \sigma_j^{h+1})] \tag{12}$$

where $p_j(t) = \frac{\pi_j^h q(x_t|\ell_j^h, m_j^h, \sigma_j^h)}{\sum_{i=1}^{K} \pi_i^h q(x_t|\ell_j^h, m_j^h, \sigma_j^h)}$. For simplicity, the Q-function is denoted by

$$Q = \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{K} p_j(t) \ln[\pi_j q(x_t|\ell_j, m_j, \sigma_j)]. \tag{13}$$

3 Proposed Fixed-Point EM Algorithm

In the EM algorithm, it is key to solve the maximization of the Q-function. We now analyze the Q-function and try to establish a fixed-point learning algorithm to solve the maximization of Q-function.

Since $\sum_{j=1}^{K} \pi_j = 1$ and ${\ell_{j1}}^2 + {\ell_{j2}}^2 = 1$ for any j, we introduce the Lagrange multipliers $\beta, \lambda_j (j = 1, \ldots, K)$ and the Lagrange function

$$L(\Theta_K, \beta, \lambda_1, \ldots, \lambda_K) = Q + \beta(1 - \sum_{j=1}^{K} \pi_j) + \sum_{j=1}^{K} \lambda_j (1 - \ell_{j1}^2 - \ell_{j2}^2). \tag{14}$$

By differentiation, we have the following derivatives:

$$\frac{\partial L}{\partial \pi_j} = \frac{1}{N}\sum_{j=1}^{K}\frac{1}{\pi_j}p_j(t) - \beta, \tag{15}$$

$$\frac{\partial L}{\partial \beta} = 1 - \sum_{j=1}^{K}\pi_j, \tag{16}$$

$$\frac{\partial L}{\partial \lambda_j} = 1 - l_{j1}^2 - l_{j2}^2, \tag{17}$$

$$\frac{\partial L}{\partial \ell_{j1}} = \frac{1}{N}\sum_{j=1}^{K}p_j(t)\frac{1}{\sigma_j^2}\{-(x_{t1}-m_{j1})[(x_{t1}-m_{j1})\ell_{j1}+(x_{t2}-m_{j2})\ell_{j2}]\}, \tag{18}$$

$$\frac{\partial L}{\partial m_{j1}} = \frac{1}{N}\sum_{j=1}^{K}p_j(t)\frac{1}{\sigma_j^2}\{-(x_{t1}-m_{j1})+\ell_{j1}(x_t-m_j,\ell_j)\}, \tag{19}$$

$$\frac{\partial L}{\partial \sigma_j} = \frac{1}{N}\sum_{j=1}^{K}p_j(t)\{\frac{1}{\sigma_j}+\frac{1}{\sigma_j^3}d^2(x_t,\ell_j,m_j)\}. \tag{20}$$

By letting these derivatives given by Eqs. (15)-(20) be 0, we have

$$\beta = \frac{1}{N}\sum_{j=1}^{K}\sum_{t=1}^{N}p_j(t), \tag{21}$$

and further obtain the following fixed-point learning algorithm:

$$m_j^{h+1} = \frac{\sum_t p_j(t)x_t}{\sum_t p_j(t)}, \tag{22}$$

$$\pi_j^{h+1} = \frac{1}{N}\sum_t p_j(t), \tag{23}$$

$$(\sigma_j^{h+1})^2 = \frac{\sum_t p_j(t)d^2(x_t,\ell_k,m_k)}{\sum_t p_j(t)}, \tag{24}$$

and ℓ_j is the eigenvector of $\Sigma_j = \sum_{j=1}^{N} p_j(t)(x_t - m_j)(x_t - m_j)^T$ corresponding to the largest eigenvalue.

Based on the above fixed-point learning algorithm, we can establish the fixed-point EM algorithm which consists of the following three steps:

(i) Initialization of the parameters.
(ii) Update m_j, π_j, σ_j^2 by Eqs. (22)-(24). Update ℓ_j by the eigenvector of $\Sigma_j = \sum_{j=1}^{N} p_j(t)(x_t - m_j)(x_t - m_j)^T$ corresponding to the largest eigenvalue.
(iii) Repeat Step (ii) until the values of parameters are unchanged.

4 Experiments Results

In this section, several simulation experiments are carried out to demonstrate the performance of the fixed-point EM algorithm for straight line detection. We consider the binary image datasets of straight lines with three kinds of noises. The true parameters of the finite mixture models for the three datasets are listed in Table 1. Obviously, the noise levels in $\mathcal{S}_2$ and $\mathcal{S}_3$ are much higher than that of $\mathcal{S}_1$.

In our experiments, we set the number of straight lines to be the true number of straight lines, i.e., $K = 4$. We implement the fixed-point EM algorithm on each dataset, with the parameters being initialized by the random Hough transform [3]. The algorithm stops if $|Q(\Theta_K^{new}) - Q(\Theta_K^{old})| < 10^{-6}$. The results of straight line detection as well as the obtained Q-function are shown in Fig. 1-3, respectively. The learned parameters of the finite mixture models on each experiment are listed in Table. 2.

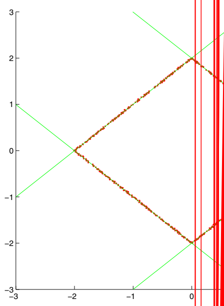
Table 1. The true parameters of the finite mixture models for the datasets $\mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{S}_3$, respectively

Sample set	π_i	ℓ_i	m_i	σ_i
$\mathcal{S}_1$	0.25	(-0.7071,0.7071)	(1,1)	0.01
	0.25	(-0.7071,0.7071)	(-1,1)	0.01
	0.25	(-0.7071,0.7071)	(-1,-1)	0.01
	0.25	(0.7071,0.7071)	(1,-1)	0.01
$\mathcal{S}_2$	0.25	(-0.7071,0.7071)	(1,1)	0.2
	0.25	(-0.7071,0.7071)	(-1,1)	0.2
	0.25	(-0.7071,0.7071)	(-1,-1	0.2
	0.25	(0.7071,0.7071)	(1,-1)	0.2
$\mathcal{S}_3$	0.25	(-0.7071,0.7071	(1,1)	0.3
	0.25	(-0.7071,0.7071	(-1,1)	0.3
	0.25	(-0.7071,0.7071	(-1,-1	0.3
	0.25	(0.7071,0.7071)	(1,-1)	0.3

It can be observed from the figures in Fig. 1 that the Q-function increases during the iterations and finally reaches its maximum. Meanwhile, the straight lines are accurately located in each case. We can also observe that the Q-function increases sharply at the beginning of the iterations. The reason may be that the parameters are initialized by the random Hough transform, being only some rough estimates of the parameters. As the initialization of the parameters becomes better, the curve of the Q-function will be more smooth.

From these experimental results, we are sure that our proposed fixed-point EM algorithm can effectively detect the straight lines in all the three datasets with different noise levels. Moreover, it is shown in Fig. 3-(c) that the fixed-point EM algorithm also performs well on the strongly noisy situation.

It is noted that the number of straight lines in the learning mixture model should be known in advance. But this information may be not available in



(a)

Fig. 1. (a). The experim
The sketch of the Q-func

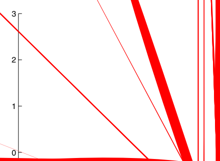


Table 2. The learned parameters of the finite mixture models on the three datasets $\mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{S}_3$, respectively

Sample set	π_i	ℓ_i	m_i	σ_i
$\mathcal{S}_1$	0.2498	(-0.7084,0.7058)	(0.9715,1.0290)	0.0103
	0.2489	(-0.7084,0.7058)	(-0.9695,1.0302)	0.0083
	0.2502	(-0.7077,0.7065)	(-0.9516,-1.0491)	0.0089
	0.2511	(0.7067,0.7075)	(1.0120,-0.9871)	0.0097
$\mathcal{S}_2$	0.2617	(0.7101,-0.7041)	(1.0192,0.9542)	0.1726
	0.2421	(0.7119, 0.7023)	(-1.0138,0.9835)	0.1812
	0.2380	(0.7210,-0.6930)	(-0.9720,-1.0285)	0.1760
	0.2582	(-0.7157,-0.6984)	(0.9778,-0.9707)	0.2138
$\mathcal{S}_3$	0.2386	(-0.7908,0.6120)	(0.8080,1.0306)	0.2695
	0.2557	(-0.7459,-0.6661)	(-0.8380,1.0016)	0.2826
	0.2335	(0.7296,-0.6839)	(-0.8854,-0.9882)	0.2867
	0.2722	(-0.7323,-0.6810)	(0.8837,-0.9461)	0.3386

practical applications. In order to overcome these weaknesses, we can introduce the Bayesian Ying-Yang (BYY) harmony learning system [17]-[18] and the entropy penalized automated model selection mechanism [19] into the fixed-point learning algorithm.

5 Conclusions

We have investigated the straight line detection problem from the finite mixture modeling. Since it is difficult to solve the maximization of the log-likelihood function, we turn to the EM algorithm and analyze the Q-function. By differentiation, we derive a fixed-point learning procedure for maximizing the Q-function and thus construct a fixed-point EM algorithm for straight line detection. It is demonstrated by the experiments that the proposed fixed-point EM algorithm can effectively locate the straight lines in a dataset.

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