An Efficien Pair i e K o i Op imi a ion Algori hm for Indeenden Componen Anal i

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A r . In the frame ork of Independent Component Anal sis (ICA), kurtosis has been used idel in designing source separation algorithms. In fact, the sum of absolute kurtosis values of all the output components is an effective objective function for separating arbitrar sources. In this paper, e propose an efficient ICA algorithm via a modified Jacobi opti-

[illegible]

2 Pair ise Optimi ation of he K o i -S m

[illegible]

$$J(\mathbf{W}) = \sum_{i=1}^{n} |\ \{y_i\}| = \sum_{i=1}^{n} |E\{(\mathbf{w}_i^T\mathbf{x})^4 - 3\}|\ , \qquad (1)$$

[illegible] $\mathbf{W}^T\mathbf{W} = \mathbf{I}$, [illegible] $\{\cdot\}$ [illegible] $E\{\cdot\}$ [illegible] $\mathbf{W}$. [illegible] $\mathbf{W}$ [illegible] $\mathbf{W}$ [illegible] (1), [illegible] $\mathbf{W}$ [illegible] l- [illegible] k- [illegible] 2- [illegible]

$$\begin{cases} \mathbf{w}_l' = \mathbf{w}_l \quad \theta + \mathbf{w}_k \ \mathrm{in}\,\theta \ , \\ \mathbf{w}_k' = -\mathbf{w}_l \ \mathrm{in}\,\theta + \mathbf{w}_k \quad \theta \ , \end{cases} \tag{2}$$

$n-2$ [illegible] $\{y_i\}$ $(i \neq l, i \neq k)$ [illegible] (1) [illegible] (l,k) [illegible] (1) [illegible]

$$J_{l,k}(\theta) = |E\{(\mathbf{w}_l'^T\mathbf{x})^4 - 3\}| + |E\{(\mathbf{w}_k'^T\mathbf{x})^4 - 3\}|$$
$$= |E\{(y_l \quad \theta + y_k \ \mathrm{in}\,\theta)^4\} - 3| + |E\{(-y_l \ \mathrm{in}\,\theta + y_k \quad \theta)^4\} - 3|. \tag{3}$$

[illegible]

$$J\ (\theta) = E\{(y_l \quad \theta + y_k \ \mathrm{in}\,\theta)^4 + (y_k \quad \theta - y_l \ \mathrm{in}\,\theta)^4\} - \quad , \tag{4}$$
$$J\ \ (\theta) = E\{(y_l \quad \theta + y_k \ \mathrm{in}\,\theta)^4 - (y_k \quad \theta - y_l \ \mathrm{in}\,\theta)^4\} \ , \tag{}$$

[illegible]

$$\theta = \quad \underset{\theta}{\quad} \quad \{|J\ (\theta)|, |J\ \ (\theta)|\} \tag{}$$
$$= \begin{cases} \quad {}_\theta |J\ (\theta)|, & \quad {}_\theta |J\ (\theta)| > \quad {}_\theta |J\ \ (\theta)|, \\ \quad {}_\theta |J\ \ (\theta)|, & \quad . \end{cases} \tag{}$$

[illegible] $J\ (\theta)$ [illegible] $J\ \ (\theta)$ [illegible]

$$J\ (\theta) = A \ \mathrm{in}(4\theta + \alpha) + c \ , \tag{}$$
$$J\ \ (\theta) = B \ \mathrm{in}(2\theta + \beta) \ , \tag{9}$$

[illegible] $A \geq 0, B \geq 0, c, \alpha, \beta$ [illegible] y_l [illegible] y_k. [illegible] (3) [illegible] (4) ( ) [illegible] ( ) (9) [illegible] $\theta = 0$ [illegible]

$$J\ (\theta)|_{\theta=0} = E\{y_l^4 + y_k^4\} - \quad = A \ \mathrm{in}(\alpha) + c \ , \tag{10}$$
$$J\ \ (\theta)|_{\theta=0} = E\{y_l^4 - y_k^4\} = B \ \mathrm{in}(\beta) \ , \tag{11}$$
$$J'(\theta)|_{\theta=0} = E\{4y_l^3y_k - 4y_k^3y_l\} = 4A \quad (\alpha) \ , \tag{12}$$
$$J'\ (\theta)|_{\theta=0} = E\{4y_l^3y_k + 4y_k^3y_l\} = 2B \quad (\beta) \ , \tag{13}$$
$$J''(\theta)|_{\theta=0} = E\{24y_l^2y_k^2 - 4y_k^4 - 4y_l^4\} = -1\ \ A \ \mathrm{in}(\alpha) \ . \tag{14}$$

fi n n $J(\theta)$ n $J(\theta)$.

$$c = \frac{3}{4}E\{y_l^4 + y_k^4\} + \frac{3}{2}E\{y_l^2 y_k^2\} - \quad , \tag{1 }$$

$$A = \sqrt{(E\{y_l^4 + y_k^4\} - \quad - c)^2 + (E\{y_l^3 y_k - y_k^3 y_l\})^2}\ , \tag{1 }$$

$$B = \sqrt{(E\{y_l^4 - y_k^4\})^2 + (2E\{y_l^3 y_k + y_k^3 y_l\})^2}\ , \tag{1 }$$

$$\alpha = \begin{cases} \ \text{n}^{-1}((E\{y_l^4 + y_k^4\} - \quad - c)/A), & \alpha > 0, \\ \pi - \ \text{n}^{-1}((E\{y_l^4 + y_k^4\} - \quad - c)/A), & , \end{cases} \tag{1 }$$

$$\beta = \begin{cases} \ \text{n}^{-1}(E\{y_l^4 - y_k^4\}/B), & \beta > 0, \\ \pi - \ \text{n}^{-1}(E\{y_l^4 - y_k^4\}/B), & . \end{cases} \tag{1}$$

n n $J(\theta)$ n $J(\theta)$, n n . (3). n . ( ), $|c| + A > B$,

$$\theta = \ \max_\theta\ |J(\theta)| = \begin{cases} (\frac{\pi}{2} - \alpha)/4, & c \geq 0, \\ (-\frac{\pi}{2} - \alpha)/4, & c < 0 \end{cases} \tag{20}$$

$$\theta = \ \max_\theta\ |J(\theta)| = (\frac{\pi}{2} - \beta)/2, \tag{21}$$

n θ . n , fi

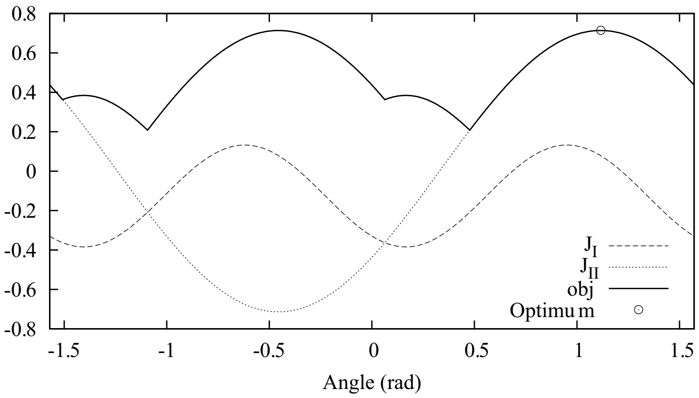
$$E\{y_l^4\},\ E\{y_k^4\},\ E\{y_l^3 y_k\},\ E\{y_l y_k^3\}\ \ \text{n}\ \ E\{y_l^2 y_k^2\} \tag{22}$$

θ. n . (22) $\mu_{4,0}$, $\mu_{0,4}$, $\mu_{3,1}$, $\mu_{1,3}$ n $\mu_{2,2}$, . n α n β n n . θ . (20), 10 . θ . (21), 11 . ( ) ffi n ".

n . 1 n (3) $J(\theta)$, $J(\theta)$ n $\mu_{4,0} = 2.\ 1$, $\mu_{0,4} = 3.0\ 3$, $\mu_{3,1} = 0.0\ 1\ 0$, $\mu_{1,3} = 0.220\ 4$ n $\mu_{2,2} = 1.0\ 0$, 4000 .

3 Pair i e K o i Op imi a ion Algori hm

n n n , $n(n-1)/2$ n n n . θ n ffi n 0 $\pm\pi/2$,



F . 1. E ample of pair ise sub-problem solution

(l, k), $y_l(t)$, $y_k(t)$, θ, $0, \pi/2$, $-\pi/2$.

$F(l, k)$, (l, k), θ, (p, q), $F(p, i)$, $F(j, q)$, $i \neq p$, $j \neq p$.

θ, $0, \pi/2$, $-\pi/2$.

INPUT $\{\mathbf{y}(t)\}$, $t = 1, \ldots, N$.

(i) *Initialize* $F(i, j) = 0$, $\mathbf{h} = (h_1, \ldots, h_n)^T = \mathbf{0}$.

(ii) $(1, 2), (1, 3), \ldots, (1, n), (2, 3), \ldots, (2, n), \ldots, (n-1, n)$. (p, q),

( ) $F(p, q) = 1$, (ii)

( ) $a(t) = y_p(t)^2$, $b(t) = y_q(t)^2$, $c(t) = y_p(t)y_q(t)$ $(t = 1, \ldots, N)$.

( ) If $h_p = 0$, $\mu_{4,0} = \sum_{t=1}^{N} a(t)^2/N$, $\mu_{4,0} = h_p$.

If $h_q = 0$, $\mu_{0,4} = \sum_{t=1}^{N} b(t)^2/N$, $\mu_{0,4} = h_q$.

$\mu_{2,2} = \sum_{t=1}^{N} c(t)^2/N$, $\mu_{3,1} = \sum_{t=1}^{N} a(t)c(t)/N$,

$\mu_{1,3} = \sum_{t=1}^{N} b(t)c(t)/N$.

( ) θ . (1)-(21). θ $|\theta| \le \pi/4$.

( ) If $|\theta| < \theta$, ( ).

(f) $\{y_p(t), y_q(t)\}' = \{y_p(t) \quad \theta + y_q(t) \sin\theta,$
$-y_p(t) \sin\theta + y_q(t) \quad \theta\}$ $(t = 1, \ldots, N)$.

$h_p = \mu_{4,0} \quad ^4\theta + 4\mu_{3,1} \quad ^3\theta \sin\theta + \mu_{2,2} \quad ^2\theta \sin^2\theta$
$+4\mu_{1,3} \quad \theta \sin^3\theta + \mu_{0,4} \sin^4\theta - 3,$

$h_q = \mu_{4,0} \sin^4\theta - 4\mu_{3,1} \sin^3\theta \quad \theta + \mu_{2,2} \quad ^2\theta \sin^2\theta$
$-4\mu_{1,3} \sin\theta \quad ^3\theta + \mu_{0,4} \quad ^4\theta - 3.$

( ) If $|\theta| > \theta$, $F(p, i)$ $(i \neq q)$ $F(j, q)$ $(j \neq p)$.

( ) $F(p, q) = 1$.

(iii) If $F(i, j) = 1$ $j > i$, (ii).

$$E\{y_p'^4\} = E\{(y_p \quad \theta + y_q \sin\theta)^4\}, \quad E\{y_q'^4\} = E\{(-y_p \sin\theta + y_q \quad \theta)^4\} \quad (23)$$

4 Comp a ion Comple i and Sim a ion

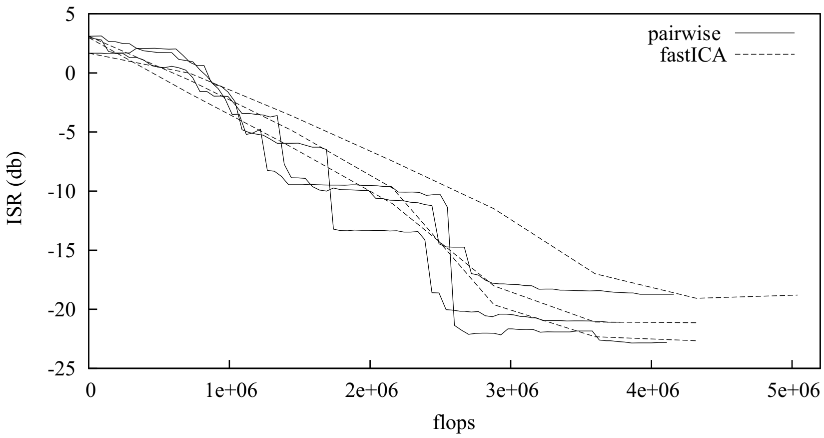
$N \gg n$, $Nn(n-1)$ $3N$ $3N$ N $4N$, $4N$

$$\mathbf{w}_i^+ = \frac{1}{N}\sum_{t=1}^{N} \mathbf{x}(t) y_i(t)^3 - 3\mathbf{w}_i , \quad (24)$$

$2nN + 2N$ $\mathbf{W}$ $n(2n+2)N$

T l 1. Source separation results from 100 Monte Carlo simulation runs

	Algorithm	ISR			Flops
		0.25 quantile	median	0.75 quantile	(average)
$n=4$	FastICA	-27.60 dB	-24.77 dB	-21.76 dB	8.92×10^5
	pair ise	-27.66 dB	-24.79 dB	-21.84 dB	6.635×10^5
$n=8$	FastICA	-21.38 dB	-19.74 dB	-18.51 dB	4.356×10^6
	pair ise	-21.48 dB	-19.88 dB	-18.51 dB	3.927×10^6
$n=16$	FastICA	-16.86 dB	-15.76 dB	-14.43 dB	2.614×10^7
	pair ise	-16.95 dB	-15.94 dB	-14.88 dB	2.207×10^7



F . 2. T pical performance curves ith 8 arbitar sources

n m l n p m n n n m n
. . n n m l n, $N=$ 000 mpl n
m (n) n f fll n n (1) n m
n, (2) n $\{-1,1\}$ l p h, (3) n $(2,2)$,
(4) mpl n m $p(x) = \frac{1}{2\sqrt{\pi}}e^{-(x-\sqrt{2}/2)^2} + \frac{1}{2\sqrt{\pi}}e^{-(x+\sqrt{2}/2)^2}$,
( ) pl n n, ( ) p h n n, ( ) t- n
f m , ( ) t- n f m 13. n m h
n m n m n m n l m , n
p nfl n fp n n p.

iff n m n n $n=4$, $n=$ n $n=1$, p
l . In m n, p l m n I l m
m m . 100 n l n p f m
f m n, n l 1 mm p f m n .
p n h m In f n - n l p
(I), f n n f$\mathbf{R}=$ **WA** n m n

$$I \quad (\mathbf{R}) = 10 \, l \quad {}_{10} \left\{ \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\sum_{j=1}^{n} r_{ij}^2}{m \quad {}_{1 \le j \le n} r_{ij}^2} - 1 \right) \right\} . \quad (2 \;)$$

. 2, . n.

θ 0.002 θ 0.02 .

5 Concl ion

J .

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