# An EfficiencPair ji e KACO i OpGmi aGon Algorichm for IndependencComponencAnal i

### $\_e$ e , and anwean $\$ $\star$

Department of Information Science, School of Mathematical Sciences and LMAM, Peking Universit, Beijing, 100871, China felixge@math.pku.edu.cn, jwma@math.pku.edu.cn

**Ab c** . In the frame ork of Independent Component Anal sis (ICA), kurtosis has been used idel in designing source separation algorithms. In fact, the sum of absolute kurtosis values of all the output components is an effective objective function for separating arbitrar sources. In this paper, e propose an efficient ICA algorithm via a modified Jacobi opti-

, n<sup>b</sup>e em lo ed n ene, l. ne oss<sup>b</sup>le w, so eso oh d-ind oho de mlinsind, orm, e he o<sup>b</sup>, <sup>b</sup>l dens n on (d),<sup>b</sup> s in, ed dewoher inson 2 o , m-h, le er inson 4, is sm le mlins in <sup>b</sup>ee, sl om ed hese e mensle d oheh h-o de <sup>T</sup> amehods .

Des e he n , o he d's, orm, on, heh h-o de mehods (e, 1,  $\frac{h}{l}esne, ll$ , oss ml, ns, n\$, she so e on onens, e m, ll inde enden, on on 2 o ed h, he s m o s, es o o ho de m, inls, ind d ed ml, ns (coses) s, on s in on em,  $\frac{h}{l}$ , b m, rm in hes , eo o h o de m, inl ml, n, Delosse and o b on G o osed, defl, on set, on sheme o se en all erhe so e s inls, wh heoe all cone en e o o e sol ons. Uno in el, he o e on e en e in he defl, on mode does no an ee he s me es l in s mme al (sml, neo s set, on) om.

 $\begin{array}{c} (11), & smoes nfi, n o ons de hesmme (lse, on o heso es Ino e o swo, , we nl ed hesmo (bsd e, les o oses o he o om onens, s, on s o obe e n on o wh ened obse, ons e eeed (s) oss-smobe e n on nd o ed h, h, snos o sm, xm, o wo-so em x n objem. I, (11), oe, nd h (lsos ded h sobe e n on nd o osed, sel-d em xm, on l o hm, wh h (mee ed he dem x n m, x, l, n o, ons) b w s oo om l, ed o m lemen, on. \\ \end{array}$ 

In h s . e, we o ose . n effi en . w se .  $o^{b}$  o m . on l o hm o I  $\downarrow$  . modfied .  $o^{b}$  o ed eon he . os s-s m  $o^{b}$  e . n on \_o m x m n he s m o . bsd e . l es o . oses o . n . o o om onen s, he oblem s ed ed ofind n . no m, l o. on . n le, wh h . n besd edd e l om d . dlow n he .  $o^{b}$  o ed e , he . os ss m  $o^{b}$  e . e n on . n be m x m ed b . se es o l n o . ons . o s . e he om . on os , we m e some modfit ons on . he s m l on ex e mens \$ n ow h. o . o osed I  $\downarrow$  l o hm s moe om . on ll effi en h . n he . s I  $\downarrow$  . l o hm nde hes mese . on e om . ne . In hese el, m x m n he . os s-s m  $o^{b}$  e . e n on n he . w se o ess n mode s n od ed n e on 2. In e on 3, we esen he . w se . os so m . on l o hm o I  $\downarrow$  n he om o hemodfied .  $o^{b}$  o m . on o ed e. hen n e on 4 he om . on om lex s . n l ed . nd om . ed w h he . s I  $\downarrow$  . l o hm wh h d owe nonl ne . s m l, on ex e mens. efinll . e, b e onl s on n e on .

# 2 Pair i e OpGmi aGon of the KAGO i -SAn

o e o m so e set, on om when end  $o^{bse}$ , on x, we e o show osed, os s sw h n l o h m, o m x m e h e, os s-s m  $o^{b}$  e e in on,

$$J(\mathbf{W}) = \sum_{i=1}^{n} | \{y_i\} | = \sum_{i=1}^{n} |E\{(\mathbf{w}_i^T \mathbf{x})^4 - 3\}| , \qquad (1)$$

95

#### 96 F. Ge and J. Ma

nde he ons in  $\mathbf{W}^T \mathbf{W}$  I, whee  $\{\cdot\}$  and  $E\{\cdot\}$  denoehe oss index e iono i indom i ble, es e el e ios lo hm wis iden-s le, index is emisibe l'ed o e in heo ho on l'o  $\mathbf{W}$ . in e  $\mathbf{W}$  so ho on ll'ons ined, sin l'en ie o ons de i wise o milion, e, onl'wo ows o  $\mathbf{W}$  is d'ed in e his e b i ens o on i his o ed e silso nown si ob lo hm, indill is o ows misibe o essed e e ed , in lin o milisters e hed.

h.n.s o he sm le dd e s eo (1), when only wo lows o  $\mathbf{W}_{\cdot}$  e d ed inde he o hio on lons ..., h. s, when he *l*-h ind *k*-h lows e o ed in 2-D lone.

$$\begin{cases} \mathbf{w}_{l}^{\prime} & \mathbf{w}_{l} \quad \mathrm{os} \, \theta + \mathbf{w}_{k} \, \mathrm{s} \, \mathrm{n} \, \theta \\ \mathbf{w}_{k}^{\prime} & -\mathbf{w}_{l} \, \mathrm{s} \, \mathrm{n} \, \theta + \mathbf{w}_{k} \quad \mathrm{os} \, \theta \end{cases},$$

$$\tag{2}$$

$$J_{l,k}(\theta) = |E\{(\mathbf{w}_l^{T}\mathbf{x})^4 - 3\}| + |E\{(\mathbf{w}_k^{T}\mathbf{x})^4 - 3\}| \\ |E\{(y_l \ \text{os}\ \theta + y_k \ \text{s}\ \text{n}\ \theta)^4\} - 3| + |E\{(-y_l \ \text{s}\ \text{n}\ \theta + y_k \ \text{os}\ \theta)^4\} - 3|. (3)$$

h s . n . on s . o<sup>b</sup> . o s l e ew se smooth. . o find . n . n l so l . o n o s m , x m m , we define

$$J(\theta) = E\{(y_l \ \operatorname{os} \theta + y_k \, \operatorname{s} \, \operatorname{\mathfrak{a}} \theta)^4 + (y_k \ \operatorname{os} \theta - y_l \, \operatorname{s} \, \operatorname{\mathfrak{a}} \theta)^4\} - \mathsf{G} , \qquad (4)$$

$$J(\theta) = E\{(y_l \ \text{os}\ \theta + y_k \ \text{s}\ \text{n}\ \theta)^4 - (y_k \ \text{os}\ \theta - y_l \ \text{s}\ \text{n}\ \theta)^4\}, \qquad ()$$

. nd ons de . n le n . e o m\_

$$\theta \quad \underset{\theta}{} m_{\theta} \mathbf{x} m_{\varepsilon} \mathbf{x} \{ |J(\theta)|, |J(\theta)| \}$$
(G)

$$\begin{cases} & \text{m}_{x} \mathbf{x}_{\theta} | J(\theta) |, \quad \text{m}_{x} \mathbf{x}_{\theta} | J(\theta) | > \text{m}_{x} \mathbf{x}_{\theta} | J(\theta) |, \\ & \text{m}_{x} \mathbf{x}_{\theta} | J(\theta) |, \quad \text{oh e w se} \end{cases}$$

I as o h.  $J(\theta)$ , ad  $J(\theta)$ , e s s a so d l a const solows.

$$J(\theta) \quad A s_{0}(4\theta + \alpha) + c , \qquad ()$$

$$J(\theta) = B s_{n}(2\theta + \beta) , \qquad ()$$

when  $h \in [0, B] \ge 0, c, \alpha, \beta$  be in the connected of  $y_l$  and  $y_k$ . So the when the extension of (3) are ended by the same tension of the set  $y_k$  are the extension of the same tension of tension of the same tension of tensio

$$J(\theta)|_{\theta=0} \quad E\{y_l^4 + y_k^4\} - \zeta \quad A s_{\mathfrak{n}}(\alpha) + c \quad , \tag{10}$$

$$J_{(\theta)}|_{\theta=0} = E\{y_l^4 - y_k^4\} = B s_{0}(\beta) , \qquad (11)$$

$$J'(\theta)|_{\theta=0} = E\{4y_l^3y_k - 4y_k^3y_l\} = 4A \ \operatorname{os}(\alpha) \ , \tag{12}$$

$$J'(\theta)|_{\theta=0} = E\{4y_l^3y_k + 4y_k^3y_l\} = 2B \text{ os}(\beta) , \qquad (13)$$

$$J''(\theta)|_{\theta=0} = E\{24y_l^2y_k^2 - 4y_k^4 - 4y_l^4\} = -1\mathsf{G}A\,s\,\mathfrak{n}(\alpha) \ . \tag{14}$$

he, bo efice , ons, eeno h o de emine  $J(\theta)$ ,  $\operatorname{nd} J(\theta)$ . he , mees n<sup>b</sup>esd ed s ollows

$$c \quad \frac{3}{4}E\{y_l^4 + y_k^4\} + \frac{3}{2}E\{y_l^2 y_k^2\} - \mathbf{G} , \qquad (1)$$

$$4 \quad \sqrt{(E\{y_l^4 + y_k^4\} - \zeta - c)^2 + (E\{y_l^3 y_k - y_k^3 y_l\})^2} \quad , \tag{14}$$

$$B \quad \sqrt{(E\{y_l^4 - y_k^4\})^2 + (2E\{y_l^3y_k + y_k^3y_l\})^2} , \qquad (1)$$

$$\alpha \quad \begin{cases} s \, \mathrm{n}^{-1}((E\{y_l^4 + y_k^4\} - \zeta_{\bullet} - c)/A), & \mathrm{os} \, \alpha > 0, \\ \pi - s \, \mathrm{n}^{-1}((E\{y_l^4 + y_k^4\} - \zeta_{\bullet} - c)/A), & \mathrm{oh} \, \mathrm{e} \, \mathrm{w} \, \mathrm{se}, \end{cases} \tag{1}$$

$$\beta \quad \begin{cases} s \, n^{-1}(E\{y_l^4 - y_k^4\}/B), & \text{os } \beta > 0, \\ \pi - s \, n^{-1}(E\{y_l^4 - y_k^4\}/B), & \text{oh e w se.} \end{cases}$$
(1)

= n ll we need o on  $e J(\theta)$  and  $J(\theta)$ , and hen lot e hem xm mo . (3). A odn o . (, ), |c| + A > B, we shold hoose

oh e w se we sh o ld h oose

hen  $\theta$  m s be here miller in the order beschemeters in smm, , e eshefie o h-o de momens

$$E\{y_l^4\}, \ E\{y_k^4\}, \ E\{y_l^3y_k\}, \ E\{y_ly_k^3\} \ \text{ad} \ E\{y_l^2y_k^2\}$$
(22)

o  $(1 \ 1)$  e  $(1 \ 1)$  best momens will be sed nse d'In he ollown we denoehe o es ondin sim le momensio (22) b  $\mu_{4,0}, \mu_{0,4}, \mu_{3,1}, \mu_{1,3}$  and  $\mu_{2,2}$ , es e d.

he is  $\alpha$  and  $\beta$  is  $\beta e$  if  $\beta e = 0$  by the iness in encoded and  $\beta$  is  $\beta e = 1$ .  $I \theta$  side emined (20), he sol ion se (len o he es m o 10.  $I \theta$  s de em ned (21), se len ohe k es m o 11. Vowe e, ad k es m o sweede ed nde wo-so em x n Fowee, and A Fowere, and  $\mathbf{A}$  es m, os we ede ed ande wo-so em x a se a solonh, ens obe, h b do hese woes m, os, ad . (.) se es some sw h a meh, a sm. Unleo e os l o hm., he e s no need o e 1 , e h e sw h n oeffi en s" se , e .

o demonst e, we see h n = -1 he  $o^{b}$  e e in on (3), swell s  $J(\theta), J(\theta)$ , ad here, here h ed or  $m_{\rm c}$  here, or h with which we have h or h or h or h and h or h o so  $em \times n$  obtem.  $h e s m lemomen s we e \mu_{4,0} = 2.6166, \mu_{0,4} = 3.03$ ,  $\mu_{3,1} = 0.0$ G1 0G,  $\mu_{1,3} = 0.220$  4 and  $\mu_{2,2} = 1.0$  0, and lead since the 4000 km state of the sta s m les.

### Pair ji e KAÇO i OpÇmi aÇon AlgoriÇhm 3

-o n o on one of s,  $ll = o^b$  swee on s s = n n(n-1)/2 of on em so ll so om onen s l $\theta$  sno s ffi en lose o lo  $\pm \pi/2$ ,

F. Ge and J. Ma



F .1. E ample of pair ise sub-problem solution

hs, shen o, ed  $[s, nd, d, o^{b}]$  o hm e oms,  $o^{b}$  swees one, e, nohe,  $n l no^{-}$ ,  $h, s^{b}$ een o, ed n one swee.

\_\_o m × m \_n he \_\_oss-s m o<sup>b</sup>e /e \_n on / \_\_ w se \_oess n  $n^{b}$ , h mode, h e o e ll om ( on os s lmos h e os o (1, 1, ..., n)hes mlemomens adm, a h a es (o, ons) o hed, hes ad d o<sup>b</sup> lohmhs.no<sup>b</sup> osd.w<sup>b</sup> , when om n , l (l,k), henchehed,  $y_l(t)$  no hed,  $y_k(t)$ h, s<sup>b</sup>eenh, nedsnehe les o, on o hs, hs, s, le, d o m, l, nd, l, n $\theta$  m s eld 0,  $\pi/2$  o  $-\pi/2$ , where here is seless.

.0 , ods h inneress, l l, ons, we can see that F(l,k) is each (l,k) and here  $b^{b}$  errors of on the matrix here l is the dimension of the set o s done", do no  $(1 \ l, e \theta \text{--oh} e w sem)$  h s flores done", ind on in e  $s \in \mathbb{R}$  hence c of on s led of (p,q), let  $\mathbb{R}$  s F(p,i) $\operatorname{nd} F(j,q)$  (o  $\lim i / p, j / p$ ) be set so on fiers some one s. heemaa eonohe, o<sup>b</sup>, lohm<sup>b</sup>eomeswheahefi, so, ll  $(s, em, ed, s done", h smodfi, on does no h, n e h e <math>{}^{b}$ eh, o o he o<sup>b</sup> lo hm.

In sml on exe men swelso o<sup>b</sup>se edh, shh o, on snole o, ffe\_ohe\_, sh, , e, o\_m, ls, e, h\_swe, dded, nhe\_s\_\_le o hessesome om consos, who sinficinde dono sec-, on 1 , h s less h,  $\theta$  s no , on  $0, \pi/2$  o  $-\pi/2$  (b) or on ssll needed), do no le 🚬 n fl s. osmm e, o m on lo hm he modified ob orde  $n^{b}$ elsed s dlows

 $INPUT_{\mathbf{x}}$  where  $\mathbf{x}$  on  $\mathbf{d} \in \{\mathbf{y}(t)\}, t = 1, \dots, N.$ 

- () Initialize, se h e fl s F(i,j) = 0, se h  $(h_1,\ldots,h_n)^T = \mathbf{0}$ .
- () e om  $\ldots$  o<sup>b</sup> swee ho h e h  $\ldots$  in h e o de  $(1,2),(1,3),\ldots,$  $(1, n), (2, 3), \dots, (2, n), \dots, (n - 1, n)$ . \_o e h (p, q), do
  - () I F(p,q) = 1, e n os e (), nd sele h e nex .

98

(b) (1 1, e  $a(t) = y_p(t)^2$ ,  $b(t) = y_q(t)^2$ ,  $c(t) = y_p(t)y_q(t)$  (t = 1, ..., N). () I  $h_p = 0$ , (1 1, e  $\mu_{4,0} = \sum_{t=1}^{N} a(t)^2/N$ , oh e wise se  $\mu_{4,0} = h_p$ . I  $h_q = 0$ , (1 1, e  $\mu_{0,4} = \sum_{t=1}^{N} b(t)^2/N$ , oh e wise se  $\mu_{0,4} = h_q$ . (1 1, e  $\mu_{2,2} = \sum_{t=1}^{N} c(t)^2/N$ ,  $\mu_{3,1} = \sum_{t=1}^{N} a(t)c(t)/N$ ,  $\mu_{1,3} = \sum_{t=1}^{N} b(t)c(t)/N$ . (d) (b)  $a \theta = 0$  of a = 0 is: (1)-(21). A d is  $\theta$  is b = h = 1.  $|\theta| \le \pi/4$ . (e) I  $|\theta| < \theta_m$ , o = o = h. (f)  $o = e = h = d = (y_p(t), y_q(t))' = \{y_p(t) = \cos\theta + y_q(t) \le a\theta, -y_p(t) \le a\theta + y_q(t) = \cos\theta\}$  (t = 1, ..., N). (l)  $a = h_p = \mu_{4,0} = \cos^4\theta + 4\mu_{3,1} = \cos^3\theta \le a\theta + 6\mu_{2,2} = \cos^2\theta \le a^2\theta + 4\mu_{1,3} = \cos^3\theta + \mu_{0,4} \le a^4\theta - 3$ , (i) I  $|\theta| > \theta$ , b = a = 1 for s F(p, i) (i / q) = a d F(j, q) (j / p). (j) e = F(p, q) = 1.

()  $\mathsf{I} F(i,j) = 1$  of  $\mathfrak{ll} j > i, s$  of  $\mathfrak{h} e$  wise of o().

In het boet o hm,  $\theta_m$  she het hold three other series of on, that  $\theta > \theta_m$  endes o het site here o holds the data of the series of the ser

 $E\{y_p^{\prime 4}\} = E\{(y_p \circ \theta + y_q \circ \theta)^4\}, \quad E\{y_q^{\prime 4}\} = E\{(-y_p \circ \theta + y_q \circ \theta)^4\} \quad (23)$ 

# 4 Comp CaCon Comple iC and Sim AaGon

ow we had e he off to on off lex o o cosed lohm. he off to not seed, ed in flow (flowing on the order of the

 $\_o$  om , son, we lso ons de he , mos $\_$ ,  $s^{\intercal}$ , lo hm  $\_$  .nsmme modew h he h d owe nonline. he om , on os  $o \_$ ,  $s^{\intercal}$ , s lm os s en on he fixed on e , on se ,

$$\mathbf{w}_{i}^{+} = \frac{1}{N} \sum_{t=1}^{N} \mathbf{x}(t) y_{i}(t)^{3} - 3\mathbf{w}_{i} ,$$
 (24)

when here 2nN + 2N floss of the whole  $m \times W = e s n(2n + 2)N$  floss he is J = s I. I ohm sill in one endess him wenter only when side endess him we have efficient.

#### 100 F. Ge and J. Ma

	Algorithm	ISR			Flops
		0.25 quantile	median	0.75 quantile	(average)
n = 4	FastICA	-27.60 dB	-24.77 dB	-21.76 dB	$8.92 \times 10^5$
	pair ise	-27.66 dB	-24.79 dB	-21.84 dB	$6.635 \times 10^{5}$
n = 8	FastICA	-21.38 dB	-19.74 dB	-18.51 dB	$4.356 \times 10^{6}$
	pair ise	-21.48 dB	-19.88 dB	-18.51 dB	$3.927 \times 10^{6}$
n = 16	FastICA	-16.86 dB	$-15.76~\mathrm{dB}$	-14.43 dB	$2.614 \times 10^7$
	pair ise	-16.95  dB	-15.94 dB	-14.88 dB	$2.207 \times 10^{7}$

 ${\bf T}$  b e 1. Source separation results from 100 Monte Carlo simulation runs



F .2. T pical performance curves ith 8 arbitar sources

e ond ed sm l. on exemens s.n. indoml ene. ed d. s so estioned have been estimated and be an

e es ed h ee diffe en dimensions when n = 4, n = ... ad n = 1, es eel Ine h sml, on, he wise lo hm and he s lo hm we es ed wh hes mem x = d = ... 100 on e lo inswere e o med o e h dimension, ad the 1 s mm, es he set, ed e om nes he set, on al smessed by et elle een e-o al owe o (I), wh h s, a on o R WA a hesml, on

$$\mathbf{I} \quad (\mathbf{R}) \quad 10 \mathbf{l}_{0-10} \left\{ \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\sum_{j=1}^{n} r_{ij}^{2}}{\mathbf{m}_{k} \mathbf{x}_{1 \le j \le n} r_{ij}^{2}} - 1 \right) \right\} \quad . \tag{2}$$

, l e om, n e sweesehed n = 2, o h ee d ffe en sm l, - onswh n.

hese exement ests we e  $o^b$  ned when  $\theta_m = 0.002$  and  $\theta = 0.02$ . hese is on the world have elmost dentition of the model of the matrix dentities dentiti

### 5 Concl $\mathbf{A}$ ion

ehve oosed, wse osso m, on, ohhohhe, obo m, on oed e om, xm ehe oss-sm obe en on o 1 A. hss.nexenson oo e oswo he losed om sol on o heo m, lo, on n lem, eshe wseo m, on oss he ndee e he modfi, on o hes, nd d, ob sheme, ns, e, o on o he om, on os, m l, on es ls onfim h, o o osed lo hm sn me ll effien, om, he o he st A. ndm, he se h. n.

### Ackno

h swo. w s s o ed b h e h  $\mathbf{D}$  o m s o nd on o n s o d on o h n o in 200 0001042.

### Reference

- 1. Cardoso, J.F.: Blind Signal Separation: Statistical Principles. Proceedings of the IEEE 86, 2009 2025 (1998)
- Comon, P.: Independent Component Anal sis a Ne Concept? Signal Processing 36, 287–314 (1994)
- Bell, A., Sejno ski, T.: An Information-Ma imi ation Approach to Blind Separation and Blind Deconvolution. Neural Computation 7, 1129 1159 (1995)
- 4. Amari, S.I., Cichocki, A., Yang, H.: A Ne Learning Algorithm for Blind Separation of Sources. Advances in Neural Information Processing 8, 757–763 (1996)
- Cardoso, J.F.: High-order contrasts for Independent Component Anal sis. Neural Computation 11, 157–192 (1999)
- Delfosse, N., Loubaton, P.: Adaptive Blind Separation of Independent Sources: a Deflation Approach. Signal Processing 45, 59–83 (1995)
- Ge, F., Ma, J.: Anal sis of the Kurtosis-Sum Objective Function for ICA. In: Sun, F., Zhang, J., Tan, Y., Cao, J., Yu, W. (eds.) ISNN 2008, Part I. LNCS, vol. 5263, pp. 579–588. Springer, Heidelberg (2008)
- Moreau, E., Macchi, O.: High-order Contrast for Self-adaptive Source Separation. International Journal of Adaptive Control and Signal Processing 10, 19–46 (1996)
- H varinen, A.: Fast and Robust Fi ed-point Algorithms for Independent Component Anal sis. IEEE Trans. Neural Net orks 10, 626–634 (1999)
- Zar oso, V., Nandi, A.K.: Blind Separation of Independent Sources for Virtuall an Source Probabilit Densit Function. IEEE Trans. Signal Processing 47, 2419 2432 (1999)
- Zar oso, V., Nandi, A.K., Herrmann, F., Millet-Roig, J.: Combined Estimation Scheme for Blind Source Separation ith Arbitar Source PDFs. Electronic Letters 37, 132–133 (2001)