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From Gaussian Processes to the Mixture of Gaussian Processes A Survey

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Abstract Gaussian process (GP) model is a paradigmatic machine learning model that combines the advantages of both kernel learning method and Bayesian inference mechanism and thus has become a very popular area in machine learning in recent years. As an extension of the GP model the Mixture of Gaussian Processes (MGP) fits datasets more effectively and thus it has a better ability of learning and generalization. However

$$t_N \propto P \mathbf{f}_{N+1} | \mathbf{X}_{N+1} \prod_{n=1}^{N+1} P t_n | f \mathbf{x}_n \quad P \mathbf{f}_{N+1} |$$

$$\mathbf{X}_{N+1} \propto \exp\left(-\frac{1}{2} \mathbf{f}_{N+1}^T \mathbf{K}_{N+1}^{-1} \mathbf{f}_{N+1}\right)$$

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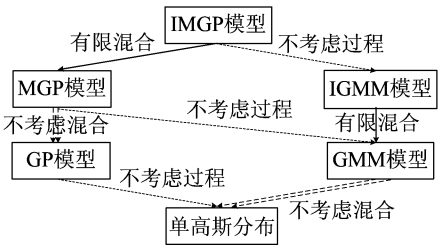
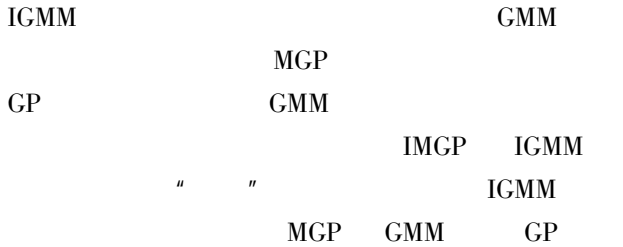
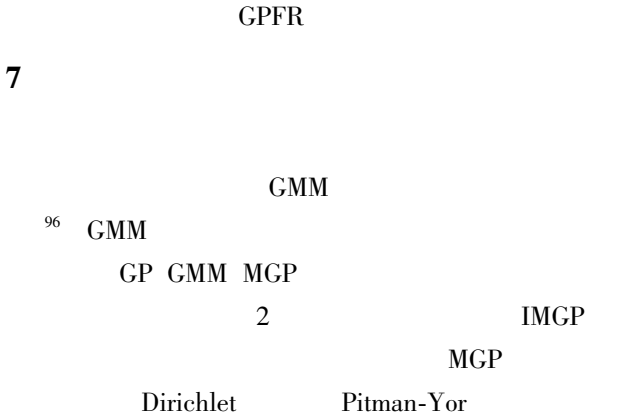
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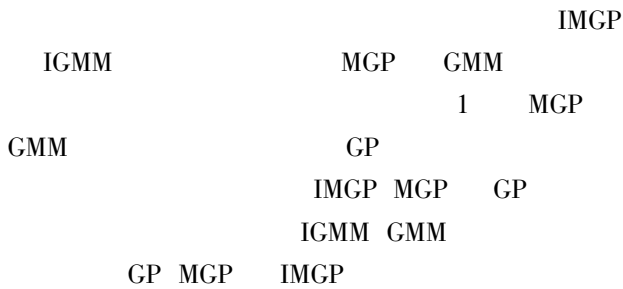
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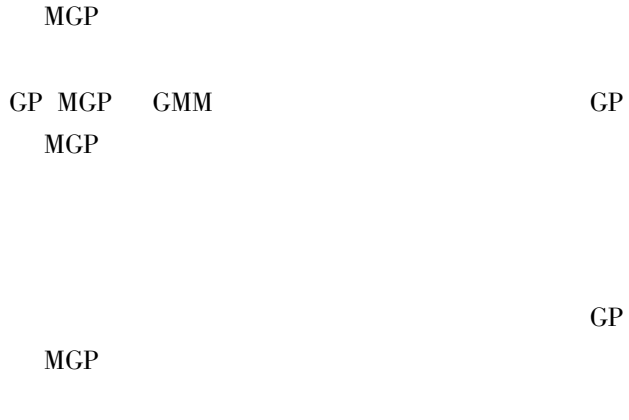
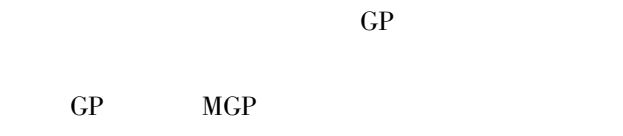


2 GP MGP GMM

Fig. 2 The hierarchical relationship of GP MGP and GMM models where the arrows denote the direction of degeneration



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