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1 I tr d ct

The allocation of wealth among alternative assets is one of an individual s most important financial decisions. The groundbreaking work of Markowit (1952) in mean-variance theor , used to anal e asset allocation, has remained the cornerstone of modern portfolio theor . This has led to numerous breakthroughs in financial economics, including the famous Capital Asset Pricing Model (Sharpe 1964; Lintner 1965a, b; Tre nor 1965; Mossin 1966). The influence of this paradigm goes far be ond academia, however. For example, it has become an integral part of modern investment management practice (Reill and Brown 2011).

More recentl , economists have also adopted the use of evolutionar principles to understand economic behavior, leading to the development of evolutionar game theor (Ma nard Smith 1982); the evolutionar implications of probabilit matching (Cooper and Kaplan 1982), group selection (Zhang et al. 2014a), and cooperation and altruism (Alexander 1974; Hirshleifer 1977, 1978); and the process of selection of firms (Luo 1995) and traders (Blume and Easle 1992; Kogan et al. 2006a; Hirshleifer and Teoh 2009). The Adaptive Markets H pothesis (Lo 2004, 2017) provides economics with a more general evolutionar perspective, reconciling economic theories based on the Efficient Markets H pothesis with behavioral economics. Under this h pothesis, the neoclassical models of rational behavior coexist with behavioral models, and what had previousl been cited as counterexamples to rationalit _ loss aversion, overconfidence, overreaction, and other behavioral biases_ become consistent with an evolutionar model of human behavior.

The evolutionar perspective brings new insights to economics from be ond the traditional neoclassical realm, helping to reconcile inconsistencies in behavior between *Homo economicus* and *Homo sapiens* (Kahneman and Tversk 1979; Brennan and Lo 2011). In particular, evolutionar models of behavior provide important insights into the biological origin of time preference and utilit functions (Rogers 1994; Waldman 1994; Samuelson 2001; Zhang et al. 2014b), in fact justif ing their existence, and allow us to derive conditions about their functional form (Hansson and Stuart 1990; Robson 1996, 2001a) (see Robson 2001b; Robson and Samuelson 2009 for comprehensive reviews of this literature). In addition, the experimental evolution of biological organisms has been suggested as a novel approach to understanding economic preferences, given that it allows the empirical stud of preferences b placing organisms in specificall designed environments (Burnham et al. 2015).

The evolutionar approach to investing is closel related to optimal portfolio growth theor , as explored b Kell (1956), Hakansson (1970), Thorp (1971), Algoet and Cover (1988), Browne and Whitt (1996), and Aurell et al. (2000), among others. While the evolutionar framework tends to focus on the long-term performance of a strateg , investors are also concerned with the short to medium term (Browne 1999). M opic investor behavior has been documented in both theoretical and empirical studies (Strot 1955; Stro an 1983; Thaler et al. 1997; Bushee 1998). Since much of the field of population genetics focuses on short-term competition between different t pes of individuals, population geneticists have applied their ideas to portfolio theor (Frank 1990, 2011; Orr 2017), in some cases considering the maximi ation of one-period expected wealth.

The market d namics of investment strategies under evolutionar selection have been explored under the assumption that investors will tr to maximi e absolute wealth (Evstigneev et al. 2002; Amir et al. 2005; Hens and Schenk-Hopp 2005; Evstigneev et al. 2006). Some studies have found that individual investors with more accurate beliefs will accumulate more wealth, and thus dominate the econom (Sandroni 2000, 2005), while others have argued that wealth d namics need not lead to rules that maximi e expected utilit using rational expectations (Blume and Easle 1992), and that investors with incorrect beliefs ma drive out those with correct beliefs (Blume and Easle 2006). Research on the performance of rational versus irrational traders has also adopted evolutionar ideas; for example, it has been shown that irrational traders can survive in the long run, resulting in prices that diverge from fundamental values (Long et al. 1990, 1991; Biais and Shadur 2000; Hirshleifer and Luo 2001; Hirshleifer et al. 2006; Kogan et al. 2006b; Yan 2008).

Traditional portfolio growth theor has focused on absolute wealth and the Kell criterion (Kell 1956; Thorp 1971). Instead of stud ing the growth of absolute wealth, however, we will consider the relative wealth in the spirit of Orr (2017). Relative wealth or income has been discussed in a number of studies (Robson 1992; Bakshi and Chen 1996; Corneo and Jeanne 1997; Hens and Schenk-Hopp 2005; Frank 2011), and the behavioral economics literature provides voluminous evidence that investors sometimes assess their performance relative to a reference group (Frank 1985; Clark and Oswald 1996; Clark et al. 2008). It is particularl important to understand the consequences of investment decisions in a setting where relative wealth is the standard, not absolute wealth. As Burnham et al. (2015) pointed out, if people are envious b caring about relative wealth, then free trade ma make all parties richer, but ma cause envious people to be less happ . If economics misunderstands human nature, then free trade ma simultaneousl increase wealth and unhappiness.

In this paper, we compare the implications of maximi ing relative wealth to maximi ing absolute wealth over both short-term and long-term investment hori ons. We use ideas from Orr (2017), and compare his results to an extension of the binar choice model of Brennan and Lo (2011). We consider two assets in a discrete-time model, and an investor who allocates her wealth between the two assets. Rather than maximi ing her absolute wealth, the investor maximi es her wealth relative to another investor with a fixed behavior. We consider the cases of one time period, multiple periods, and an infinite time hori on. We then ask the question: what is the optimal behavior for an investor as a function of the environment, given that the environment consists of the asset returns and the behavior of the other participants? In our approach, we define relative wealth as a proportion of the total wealth, which corresponds closel to the allele frequenc in population genetics. This analog acts as a bridge to earlier literature on the relevance of relative wealth to behavior. While some of our results will be familiar to population geneticists, the do not appear to be widel known in a financial context. For completeness, we derive them from first principles in this new context.

Our approach leads to several interesting conclusions about the Kell criterion. We show that it is the optimal behavior if the investor maximi es her absolute wealth in the case of an infinite hori on (see also Brennan and Lo 2011). In the case that the investor maximi es her relative wealth, we identif the conditions under which it is optimal, and the conditions under which the investor should deviate from it. The investor s initial relative wealth_ which represents the investor s market power_ pla s a critical role. Moreover, the dominant investor s optimal behavior is different from the minorant investor s optimal behavior.

In Sect. 2 of this paper, we consider a two-asset model in which investors maximi e their absolute wealth. It is shown that the long-run optimal behavior is equivalent to the behavior implied b the Kell criterion. Section 3 extends the binar choice model, and considers in a non-game-theoretic framework the case of two investors who maximi e their wealth relative to the population, given the other investor s behavior. The Kell criterion emerges as a special case under certain environmental conditions. Section 4 provides a numerical example to illustrate the theoretical results. Section 5 discusses several implications which can be tested through experimental evolutionar techniques. We end with a discussion in Sect. 6, and provide proofs in Appendix A.

2 Ma z ab te wea t : t e Ke y cr ter

Consider two assets, a and b, in a discrete-time model, each generating gross returns $X_a \in (0, \infty)$ and $X_b \in (0, \infty)$ in one period. For example, asset a can be a risk asset whereas asset b can be the riskless asset. In this case, $X_a \in (0, \infty)$ and $X_b = 1 + r$, where r is the risk-free interest rate. In general, $(X_{a,t}, X_{b,t})$ are IID over time $t = 1, 2, \ldots$, and are described b the probabilit distribution function $\Phi(X_a, X_b)$.

Consider an investor who allocates $f \in [0, 1]$ of her wealth in asset a and $1 - f$ in asset b. We will refer to f as the investor s behavior henceforth. We assume that:

A t 1 (X_a, X_b) and $\log(fX_a + (1-f)X_b)$ have finite moments up to order 2 for all $f \in [0, 1]$.

Note that Assumption 1 guarantees that the gross return of an investment portfolio is positive. In other words, the investor cannot lose more than what she has. This is made possible b assuming that X_a and X_b are positive and f is between 0 and 1. In other words, the investor onl allocates her mone between two assets, and no short-selling or leverage is allowed.[1]

Let n_t^f be the total wealth of investor f in period t. To simplif notation, let $\omega_t^f = fX_{a,t} + (1-f)X_{b,t}$ be the gross return of investor f s portfolio in period t. With these notational conventions in mind, the portfolio growth from period $t-1$ to period t is:

$$n_t^f = n_{t-1}^f \left(fX_{a,t} + (1-f)X_{b,t} \right) = n_{t-1}^f \omega_t^f .$$

[1] One could relax this assumption b allowing short-selling and leverage, which corresponds to $f < 0$ or $f > 0$. However, f still needs to be restricted such that $fX_a + (1-f)X_b$ is alwa s positive. This does not change our results in an essential wa , but it will complicate the presentation of some results mathematicall . Therefore, we stick to the simple assumption that $f \in [0, 1]$ as in Brennan and Lo (2011).

Through backward recursion, the total wealth of investor f in period T is given b

$$n_T^f = \prod_{t=1}^{T} \omega_t^f = \exp\left(\sum_{t=1}^{T} \log \omega_t^f\right).$$

Taking the logarithm of wealth and appl ing Kolmogorov s law of large numbers, we have:

$$\frac{1}{T} \log n_T^f = \frac{1}{T} \sum_{t=1}^{T} \log \omega_t^f \xrightarrow{p} \mathbb{E}\left[\log \omega_t^f\right] = \mathbb{E}\left[\log\left(f X_a + (1-f) X_b\right)\right], \quad (1)$$

as T increases without bound, where $\xrightarrow{p}$ in (1) denotes convergence in probabilit . We have assumed that $n_0^f = 1$ without loss of generalit .

The expression (1) is simpl the expectation of the log-geometric-average growth rate of investor f s wealth, and we will call it $\mu(f)$ henceforth:

$$\mu(f) = \mathbb{E}\left[\log\left(f X_a + (1-f) X_b\right)\right]. \quad (2)$$

The optimal f that maximi es (2) is given b

Pr t 1 *The optimal allocation f^{Kelly} that maximizes investor f s absolute wealth as T increases without bound is*

$$f^{Kelly} = \begin{cases} 1 & \text{if } \mathbb{E}[X_a/X_b] > 1 \text{ and } \mathbb{E}[X_b/X_a] < 1, \\ \text{solution to (4)} & \text{if } \mathbb{E}[X_a/X_b] \geq 1 \text{ and } \mathbb{E}[X_b/X_a] \geq 1, \\ 0 & \text{if } \mathbb{E}[X_a/X_b] < 1 \text{ and } \mathbb{E}[X_b/X_a] > 1, \end{cases} \quad (3)$$

where f^{Kelly} is defined implicitly in the second case of (3) by:

$$\mathbb{E}\left[\frac{X_a - X_b}{f^{Kelly} X_a + (1 - f^{Kelly}) X_b}\right] = 0. \quad (4)$$

The optimal allocation given in Proposition 1 coincides with the Kell criterion (Kell 1956; Thorp 1971) in probabilit theor and the portfolio choice literature. To emphasi e this connection, we refer to this optimal allocation as the Kell criterion henceforth. As we will see, in the case of maximi ing an individual s relative wealth, the Kell criterion pla s a ke role as a reference strateg .

In portfolio theor , the Kell criterion is used to determine the optimal si e of a series of bets in the long run. Although this strateg s promise of doing better than an other strateg in the long run seems compelling, some researchers have argued against it, principall because the specific investing constraints of an individual ma override

3 Ma z re at e wea t

In this section, we consider two investors. The first investor allocates $f \in [0,\ 1]$ of her wealth in asset a and $1 - f$ in asset b. The second investor allocates $g \in [0,\ 1]$ of his wealth in asset a and $1 - g$ in asset b. Investor f s objective is to maximi e the proportion of her wealth relative to the total wealth in the population, which we define as investor f s relative wealth. Note that we use f and g to mean the proportion of wealth and as a label for the investor, to simplif notation.

Here we can introduce a concept taken from evolutionar theor . In population genetics, the metric for natural selection is the expected reproduction of a genot pe divided b the average reproduction of the population, i.e., the relative reproduction, analogous to investor f s relative wealth. Our consideration of the relative wealth rather than the absolute wealth naturall unlocks existing tools and ideas from population genetics for us.

In the case of maximi ing relative wealth, the initial wealth pla s an important role in the optimal allocation. Let $\lambda \in (0,\ 1)$ be the relative initial wealth of investor f:

$$\lambda = \frac{n_0^f}{n_0^f + n_0^g}.$$

Let q_t^f be the relative wealth of investor f in subsequent periods $t = 1,\ 2, \ldots$ q_t^f and q_t^g are defined similarl :

$$q_t^f = \frac{n_t^f}{n_t^f + n_t^g} = \frac{1}{1 + n_t^g / n_t^f},$$
$$q_t^g = 1 - q_t^f.$$

It is obvious that the ratio n_t^g / n_t^f is sufficient to determine the relative wealth q_t^f. Let R_T^f be the T-period average log-relative-growth:

$$R_T^f = \frac{1}{T} \log \frac{\prod_{t=1}^T \omega_t^g}{\prod_{t=1}^T \omega_t^f} = \frac{1}{T} \sum_{t=1}^T \log \frac{\omega_t^g}{\omega_t^f}. \tag{5}$$

Then we can write the relative wealth in period T as:

$$q_T^f = \frac{1}{1 + \frac{n_T^g}{n_T^f}} = \frac{1}{1 + \frac{(1-\lambda) \prod_{t=1}^T \omega_t^g}{\lambda \prod_{t=1}^T \omega_t^f}} = \frac{1}{1 + \frac{1-\lambda}{\lambda} \exp\left(T R_T^f\right)}. \tag{6}$$

Analogs to Eqs. (5) (6) are well known in the population genetics literature, used when the fitnesses of genot pes are assumed to var randoml through time. (For reviews of this literature, see Felsenstein 1976 and Gillespie 1991, Chap. 4.)

3.1 O e- er d re t

We first consider a m opic investor, who maximi es her expected relative wealth in the first period. B (6), the expectation of q_1^f is:

$$\mathbb{E}\left[q_1^f\right] = \mathbb{E}\left[\frac{1}{1 + \frac{1-\lambda}{\lambda}\frac{\omega^g}{\omega^f}}\right].$$

Here we have dropped the subscripts in ω_1^f and ω_1^g, and instead simpl use ω^f and ω^g, because there is onl one period to consider.

Given investor g, we denote f_1^* as investor f s optimal allocation that maximi es $\mathbb{E}[q_1^f]$. There is no general formula to compute $\mathbb{E}[q_1^f]$ because it involves the expectation of the ratio of random variables. Population geneticists sometimes use diffusion approximations to estimate similar quantities, for example, the change in allele frequenc (Gillespie 1977; Frank and Slatkin 1990; Frank 2011), which are essentiall linear approximations of the nonlinear quantit using the Ta lor series. The diffusion approximation is also used b Orr (2017) in a similar model for relative wealth.

Without the diffusion approximation, one can still characteri e f

than investor g. Similarl , when investor g takes a position that is more conservative than the Kell criterion, investor f should never be even more conservative than investor g.

It is interesting to compare the optimal behavior f_1^* with the Kell criterion f^{Kelly}, which is provided in the next proposition. It shows that when g is not far from the Kell criterion, the relationship between f_1^* and f^{Kelly} depends on the initial relative wealth of investor f.

Pr t 4 *If investor f is the dominant investor $\left(\lambda > \frac{1}{2}\right)$, then she should be locally more/less risky than Kelly in the same way as investor g: for small $\epsilon > 0$,*

$$g = f^{Kelly} - \epsilon \Rightarrow f_1^* < f^{Kelly},$$
$$g = f^{Kelly} + \epsilon \Rightarrow f_1^* > f^{Kelly}.$$

If investor f is the minorant investor $\left(\lambda < \frac{1}{2}\right)$, then she should be locally more/less risky than Kelly in the opposite way as investor g: for small $\epsilon > 0$,

$$g = f^{Kelly} - \epsilon \Rightarrow f_1^* > f^{Kelly},$$
$$g = f^{Kelly} + \epsilon \Rightarrow f_1^* < f^{Kelly}.$$

If investor f starts with the same amount of wealth as investor g $\left(\lambda = \frac{1}{2}\right)$, then she should be locally Kelly:

$$g \approx f^{Kelly} \Rightarrow f_1^* \approx f^{Kelly}.$$

Note that when g is far from the Kell criterion, the conclusions in Proposition 4 ma not necessaril hold. Section 4 provides a numerical example (see Fig. 1b) where investor f is the minorant investor $\left(\lambda < \frac{1}{2}\right)$ and $g \ll f^{Kelly}$, but $f_1^* < f^{Kelly}$. However, Orr (2017) has shown that these results are still approximatel true for an g up to a diffusion approximation, which is consistent with the numerical results for maximi ing one-period relative wealth in Fig. 1a. We will provide more discussion on this point in Sect. 4.

3.2 M t- er d re . t

The previous results are based on maximi ing the expected relative wealth in period 1: $\mathbb{E}[q_1^f]$. To generali e these results to maximi ing expected relative wealth in period T: $\mathbb{E}[q_T^f]$, we have:

Pr t 5 *The optimal behavior of investor f that maximizes expected relative wealth in the Tth period is given by:*

$$f_T^* = \begin{cases} 1 & \textit{if } \mathbb{E}\left[\frac{\exp(TR_T^1)\left(T-\sum_{t=1}^T \frac{X_{bt}}{X_{at}}\right)}{\left(1+\frac{1-\lambda}{\lambda}\exp(TR_T^1)\right)^2}\right] > 0, \\ \textit{solution to } (10) & \textit{if } \mathbb{E}\left[\frac{\exp(TR_T^1)\left(T-\sum_{t=1}^T \frac{X_{bt}}{X_{at}}\right)}{\left(1+\frac{1-\lambda}{\lambda}\exp(TR_T^1)\right)^2}\right] \leq 0 \textit{ and } \mathbb{E}\left[\frac{\exp(TR_T^0)\left(\sum_{t=1}^T \frac{X_{at}}{X_{bt}}-T\right)}{\left(1+\frac{1-\lambda}{\lambda}\exp(TR_T^0)\right)^2}\right] \geq 0, \\ 0 & \textit{if } \mathbb{E}\left[\frac{\exp(TR_T^0)\left(\sum_{t=1}^T \frac{X_{at}}{X_{bt}}-T\right)}{\left(1+\frac{1-\lambda}{\lambda}\exp(TR_T^0)\right)^2}\right] < 0, \end{cases} \tag{9}$$

where f_1^* *is defined implicitly in the second case of* (9) *by*:

$$\mathbb{E}\left[\exp(TR_T^f)\sum_{t=1}^T \frac{X_{at}-X_{bt}}{fX_{at}+(1-f)X}\right.$$

of Sect. 4 show that the condition that g is near the Kell criterion is essential (see Fig. 2a).

3.3 I te r z

Recall from (5) that the T-period average log-relative-growth R_T^f is given b :

$$R_T^f = \frac{1}{T}\sum_{t=1}^{T}\log\omega_t^g - \frac{1}{T}\sum_{t=1}^{T}\log\omega_t^f \xrightarrow{p} \mu(g) - \mu(f), \tag{11}$$

as T increases without bound. It is therefore eas to see from (6) that:

Pr t 7 *As T increases without bound, the relative wealth of investor f converges in probability to a constant*:

$$q_T^f \xrightarrow{p} \begin{cases} 0 \text{ if } \mu(f) < \mu(g), \\ \lambda \text{ if } \mu(f) = \mu(g), \\ 1 \text{ if } \mu(f) > \mu(g). \end{cases} \tag{12}$$

Proposition 7 is consistent with well-known results in the population genetics literature (see Gillespie 1973, for example) as well as in the behavioral finance literature, as in Brennan and Lo (2011). It asserts that investor f s relative wealth will converge to 1 as long as its log-geometric-average growth rate $\mu(f)$ is greater than investor g s. This implies that when T increases without bound, there are multiple behaviors that are all optimal in the following sense:

$$\begin{aligned} \arg\max_f \lim_{T\to\infty} q_T^f &= \arg\max_f \mathbb{E}\left[\lim_{T\to\infty} q_T^f\right] \\ &= \arg\max_f \lim_{T\to\infty} \mathbb{E}\left[q_T^f\right] = \{f : \mu(f) > \mu(g)\}. \end{aligned}$$

Note that the above equalit uses the dominant convergence theorem (q_T^f is alwa s bounded) to switch the limit and the expectation operator.

However, this is not equivalent to the limit of the optimal behavior f_T^* as T increases without bound because one cannot switch the operator‘ arg max and‘ lim in general, and

$$\arg\max_f \lim_{T\to\infty} \mathbb{E}\left[q_T^f\right] \neq \lim_{T\to\infty} \arg\max_f \mathbb{E}\left[q_T^f\right].$$

In fact, Sect. 4 provides such an example.

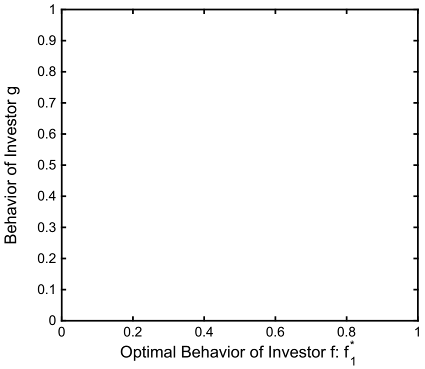
4 A er ca e a e

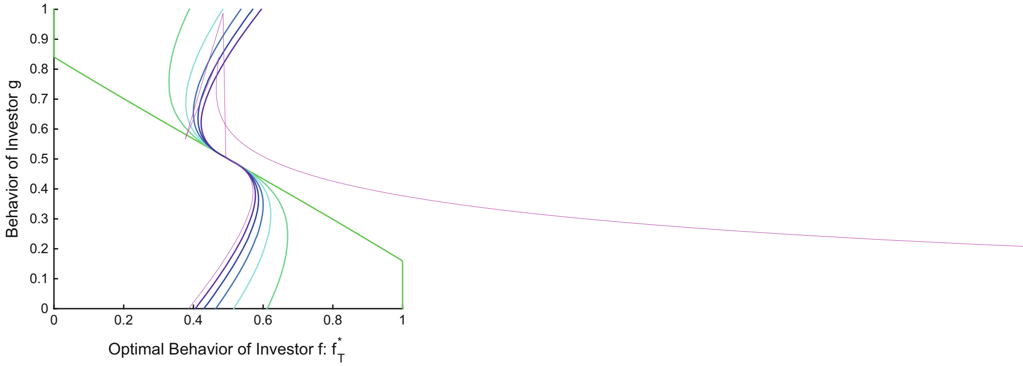
We construct a numerical example in this section to illustrate the results of Sects. 2 and 3. Consider the following two simple assets:

$$X_a = \begin{cases} \alpha \text{ with probabilit } p, \\ \beta \text{ with probabilit } 1-p, \end{cases} \qquad X_b = \gamma \quad \text{with probabilit } 1.$$

In this case, asset a is risk and asset b is riskless. The expected relative wealth of investor f in period T is explicitl given b :

$$\mathbb{E}\left[q_T^f\right] = \sum_{k=0}^{T} \qquad\qquad \binom{T}{k}$$


Behavior of Investor g
Optimal Behavior of Investor f: f_1^*
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Behavior of Investor g
Optimal Behavior of Investor f: f_T^*
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1

rather than on the absolute wealth. Relative wealth is important financiall because success and satisfaction are sometimes measured b investors relative to the success of others (Robson 1992; Frank 1990, 2011; Bakshi and Chen 1996; Clark and Oswald 1996; Corneo and Jeanne 1997; Clark et al. 2008). Our model considers the case of two investors in a non-game-theoretic framework. We show how the optimal behavior of one investor is dependent on the other investor s behavior, which might be far from the Kell criterion. While some of our results are alread known in the finance literature or the population genetics literature, the are not known together in both, and therefore the are included for completeness.

We consider m opic investors who maximi e their expected relative wealth over a single period, and investors who maximi e their relative wealth over multiple periods. Similar consequences hold in both cases. When one investor is wealthier than the other, that investor should roughl mimic the other s behavior in being more or less aggressive than the Kell criterion. Conversel , when one investor is poorer than the other, that investor should roughl act in the opposite manner of the other investor (Orr 2017).

As described above, it should be possible to design empirical biological studies to test the ideas of this paper. For example, one could design an experimental evolutionar stud with a riskless condition (with constant fitness, corresponding to a fixed pa off) and a risk condition (with variable fitness, corresponding to different pa offs), much like the numerical example considered in Sect. 4. More generall , one could design an experimental environment with two random fitnesses that follow two different distributions. B var ing the proportion of each population t pe exposed to each environment, one could create an t pe of investor as described in our model. Eventuall , one would observe the growth of different t pes of investors to test various predictions about relative wealth in this paper.

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A e d A: Pr f

Proof of Proposition 1 See Brennan and Lo (2011). □

Proof of Proposition 2 The first partial derivative of $\mathbb{E}[q_1^f]$ to f is:

$$\frac{\partial \mathbb{E}[q_1^f]}{\partial f} = \lambda(1-\lambda)\mathbb{E}\left[\frac{(X_a - X_b)\omega^g}{(\lambda\omega^f + (1-\lambda)\omega^g)^2}\right].$$

The second partial derivative of $\mathbb{E}[q_1^f]$ to f is:

$$\frac{\partial^2 \mathbb{E}[q_1^f]}{\partial f^2} = -2\lambda^2(1-\lambda)\mathbb{E}\left[\frac{(X_a - X_b)^2\omega^g}{(\lambda\omega^f + (1-\lambda)\omega^g)^3}\right] \leq 0,$$

which indicates that $\mathbb{E}[q_1^f]$ is a concave function of f. Therefore, it suffices to consider the value of the first partial derivative at its endpoints 0 and 1.

$$f_1^* = \begin{cases} 1 & \text{if } \frac{\partial \mathbb{E}[q_1^f]}{\partial f}\Big|_{f=1} > 0, \\ 0 & \text{if } \frac{\partial \mathbb{E}[q_1^f]}{\partial f}\Big|_{f=0} < 0, \\ \text{solution to } \frac{\partial \mathbb{E}[q_1^f]}{\partial f} = 0 \text{ otherwise.} \end{cases}$$

Proposition 2 follows from trivial simplifications of the above equation. □

Proof of Proposition 3 Consider $\frac{\partial \mathbb{E}[q_1^f]}{\partial f}$ when $f = g$:

$$\frac{\partial \mathbb{E}[q_1^f]}{\partial f}\Bigg|_{f=g} = \lambda(1-\lambda)\mathbb{E}\left[\frac{X_a - X_b}{fX_a + (1-f)X_b}\right].$$

Note that the righthand side consists of a factor that also appears in the first order condition (4) of the Kell criterion. Therefore its sign is determined b whether f is larger than f^{Kelly}:

$$\frac{\partial \mathbb{E}[q_1^f]}{\partial f}\Bigg|_{f=g} \begin{cases} >0 \text{ if } f = g < f^{Kelly}, \\ =0 \text{ if } f = g = f^{Kelly}, \\ <0 \text{ if } f = g > f^{Kelly}. \end{cases} \tag{A.1}$$

Since $\mathbb{E}[q_1^f]$ is concave as a function of f for an g, we know that:

$$f_1^* \begin{cases} >g \text{ if } g < f^{Kelly}, \\ =g \text{ if } g = f^{Kelly}, \\ <g \text{ if } g > f^{Kelly}, \end{cases}$$

which completes the proof. □

Proof of Proposition 4 The cross partial derivative of $\mathbb{E}[q_1^f]$ is:

$$\frac{\partial^2 \mathbb{E}[q_1^f]}{\partial f \partial g} = \lambda(1-\lambda)\mathbb{E}\left[\frac{(X_a - X_b)^2(\lambda\omega^f - (1-\lambda)\omega^g)}{(\lambda\omega^f + (1-\lambda)\omega^g)^3}\right].$$

Consider $\frac{\partial^2 \mathbb{E}[q_1^f]}{\partial f \partial g}$ when $f = g = f^{Kelly}$:

$$\frac{\partial^2 \mathbb{E}[q_1^f]}{\partial f \partial g}\Bigg|_{f=g} = 2\lambda(1-\lambda)\left(\lambda - \frac{1}{2}\right)\mathbb{E}\left[\left(\frac{X_a - X_b}{fX_a + (1-f)X_b}\right)^2\right] \begin{cases} <0 \text{ if } \lambda < \frac{1}{2}, \\ =0 \text{ if } \lambda = \frac{1}{2}, \\ >0 \text{ if } \lambda > \frac{1}{2}. \end{cases}$$

The first order condition (A.1) is 0 when $f = g = f^{Kelly}$, so when g is near f^{Kelly}, the sign of the first order condition is determined b whether λ is greater than, equal to, or less than 1/2. For example, if $\lambda < 1/2$, then the derivative of the first order condition (A.1) with respect to g is negative, which implies that the first order condition is negative when $g = f^{Kelly} + \epsilon$, where ϵ is a small positive quantit . Therefore, when $g = f^{Kelly} + \epsilon$, f_1^* is smaller than f^{Kelly}. The cases when $\lambda > 1/2$ and $\lambda = 1/2$ follow similarl . □

Proof of Proposition 5 The first partial derivative of $\mathbb{E}[q_T^f]$ to f is:

$$\frac{\partial \mathbb{E}[q_T^f]}{\partial f} = \frac{1-\lambda}{\lambda} \mathbb{E} \left[\frac{\exp(T R_T^f) \sum_{t=1}^{T} \frac{X_{at} - X_{bt}}{f X_{at} + (1-f) X_{bt}}}{} \right.$$

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