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(E. A. Poe) 1931

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(University College London)

J. Neyman (1894-1981)

Fisher-Behrens [15]

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1945 J. Neyman H. Hotelling (1895-1973
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North Carolina, Chapel Hill)

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(A. Wald)
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E. L. Lehmann (1917-2009)
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"[8] 1947 H. Hotelling

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W. L. Deemer I.
Olkin \Biometrika" [7]
H. Robbins \ "
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to the Theory of Fourier Integrals

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1997

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114

(1) [15] 1938 Behrens-Fisher $X_1, X_2, \ldots, X_n$ $Y_1, Y_2, \ldots, Y_m$ $N(\mu_1, \sigma_1^2)$ $N(\mu_2, \sigma_2^2)$ σ_1 σ_2 $H_0: \mu_1 = \mu_2$ Behrens-Fisher

$$U = (\bar{X} - \bar{Y})^2/(A_1 S_1^2 + A_2 S_2^2),$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad \bar{Y} = \frac{1}{m}\sum_{i=1}^{m} Y_i,$$

$$S_1^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2, \quad S_2^2 = \sum_{i=1}^{m}(Y_i - \bar{Y})^2,$$

A_1 A_2 $A_1 = A_2 = (m+n)/[(n+m-2)nm]$ U t u_1 $A_1 = 1/(n(n-1)), A_2 = 1/(m(m-1))$ U Behrens-Fisher u_2 U $\{U > C\}$ $\theta = \sigma_1^2/\sigma_2^2$ $\lambda = (\mu_1 - \mu_2)^2/(\frac{1}{n}\sigma_1^2 + \frac{1}{m}\sigma_2^2)$ H. Scheffe (1970) " " $\lambda = 0$ θ u_1 u_2 H_0 ($n = m$) u_2 θ " " " "

(2) [16]

$$y = A\beta + \varepsilon$$

$y = (y_1, \ldots, y_n)^T$ (T) $\beta = (\beta_1, \ldots, \beta_p)^T$ A $n \times p$ (p) $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)^T$

$$E\varepsilon_i = 0, \quad E\varepsilon_i^2 = \sigma^2, \quad E\varepsilon_i^4 = \alpha_i\sigma^4 \qquad (i = 1, \ldots, n),$$

σ α_i σ $Q = Q(y_1, y_2, \ldots, y_n)$ $y_1, y_2, \ldots, y_n$ (i) β σ, $EQ = \sigma^2$; (ii) Q β (iii) (i) (ii) Q_1 Q Q_1 , Q σ^2

$Q = y^T ¤ y$ ¤ n Q $¾^2$

¤

$$¤ = M D_\tau M;$$

$M = I - A(A^TA)^{-1}A^T$ I $D_\tau = \begin{pmatrix} ¿_1 & 0 & \cdot & 0 \\ 0 & ¿_2 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & ¿_n \end{pmatrix};$ $¿_1; ¿_2; :::; ¿_n$

$$F = \sum_{i,j} ¹_{ij} ¿_i ¿_j$$

$\sum_{i=1}^{n} m_{ii} ¿_i = 1$

$$¹_{ij} = \sum_{k=1}^{n} (®_k - 3) m_{ki}^2 m_{kj}^2 + 2m_{ij}^2$$

$(i = 1; :::; n; j = 1; ::; n);$ m_{ij} $M = (m_{ij})_{n\times n}$ $®_i = ¾^{-4} E "_i^4$ $(i = 1; :::; n)$

$¾^2$

$$S_0^2 = \frac{1}{n-p} \parallel y - A\hat{¯} \parallel^2$$

($\hat{¯}$ ¯ $\parallel z \parallel$ z)

$$(n-p) \sum_{i=1}^{n} (®_i - 3) m_{ii} m_{ik}^2 = m_{kk} \sum_{i=1}^{n} (®_i - 3) m_{ii}^2:$$

(3)

[17] Hotelling T^2

T^2 $\hat{A}^2$

$\hat{A}^2$ T^2

μ_i

$W = \prod(1 - \mu_i)$ $V = \sum \mu_i = (1 - \mu_i)$

[18]

[19]

$Y_1; Y_2; :::; Y_m; Z_1; :::; Z_n$

$$p(y_1; :::; y_m; z_1; :::; z_n) = (\sqrt{2¼}¾)^{-(m+n)} exp\{-\frac{1}{2¾^2}[\sum_{i=1}^{m}(y_i - ´_i)^2 + \sum_{i=1}^{n} z_i^2]\};$$

$\eta_1, \dots, \eta_m$ m σ ( )

$$H_0 : \eta_1 = \eta_2 = \dots = \eta_{n_1} = 0,$$

n_1 m

$$F = \frac{\sum_{i=1}^{n_1} y_i^2}{\sum_{i=1}^{n_1} y_i^2 + \sum_{i=1}^{n} z_i^2},$$

$$W_0 = \{(y_1, \dots, y_{n_1}, z_1, \dots, z_n) : F \geq F_\alpha\},$$

F_α $P(\bar{F} \geq F_\alpha | H_0) = \alpha$ $\bar{F} = \sum_{i=1}^{n_1} Y_i^2 / (\sum_{i=1}^{n_1} Y_i^2 + \sum_{i=1}^{n} Z_i^2)$ W_0 H_0 λ $\beta_0(\lambda)$

$$\lambda = \frac{1}{2\sigma^2} \sum_{i=1}^{n_1} \eta_i^2.$$

W $(y_1, \dots, y_m, z_1, \dots, z_n)$ W α W λ $\beta(\lambda)$ $\lambda > 0$ $\beta(\lambda) \leq \beta_0(\lambda)$ H_0 W_0 α λ F (H. B. Mann Analysis and Design of Experiments, 1949) Simaika (1941) Hotelling T^2 Lehmann Scheffé [4] (4) 1938 1945

S $X_1, X_2, \dots, X_N$ $N(0, \Sigma)$

$$A \triangleq (N-1)S = \sum_{\alpha=1}^{N} (X_\alpha - \bar{X})(X_\alpha - \bar{X})^T \qquad (1)$$

Wishart $W(\Sigma, N-1)$ [20] Wishart [5] [21] A B Wishart $W(\Sigma, m)$ $W(\Sigma, n)$ $m \geq p, n \geq p$ (p $\sum$) $\mu_1 \geq \mu_2 \geq \dots \geq \mu_p$

$$|A - \mu(A+B)| = 0$$

$\mu_1, \ldots, \mu_p$

$$\prod_{i=1}^{p} \mu_i^{\frac{1}{2}(m-p-1)} \prod_{i=1}^{p} (1-\mu_i)^{\frac{1}{2}(n+p-1)}) \prod_{i=1}^{p} \prod_{j=i+1}^{p} (\mu_i - \mu_j).$$

(1) A §

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}; \quad § = \begin{pmatrix} §_{11} & §_{12} \\ §_{21} & §_{22} \end{pmatrix};$$

A_{11} $§_{11}$ p_1 A_{22} $§_{22}$ p_2

$$\begin{vmatrix} -¸A_{11} & A_{12} \\ A_{21} & -¸A_{22} \end{vmatrix} = 0; \quad \begin{vmatrix} -¸§_{11} & §_{12} \\ §_{21} & -¸§_{22} \end{vmatrix} = 0.$$

([35] 142-149)

A B A Wishart B Wishart

$$|A - ÁB| = 0$$

([35] 129-140)

$X_1, X_2, \ldots, X_N$

$T = Q/S$

$$Q = \sum_{i,j=1}^{N} a_{ij}(X_i - \bar{X})(X_j - \bar{X}); \quad S = \sum_{i=1}^{N} (X_i - \bar{X})^2.$$

a_{ij} N N T

([35] 224-228)

$f(\vartheta_1, \ldots, \vartheta_k)$ $\vartheta_1, \ldots, \vartheta_k$

$f(\cdot)$

(0)

(5)

[6] [22] Berry

$»_1, »_2, \ldots, »_n$ n $E»_i = 0, E»_i^2 = 1$ $(i = 1, \ldots, n)$ (

0 1)

$$\bar{»} = \frac{1}{n}\sum_{i=1}^{n} »_i; \quad ´ = \frac{1}{n}\sum_{i=1}^{n} (»_i - \bar{»})^2;$$

$©(x)$ $\bar{»}$ ´

(n) Cramér

$F_n(x) \triangleq P(\sqrt{n}\bar{»} \le x)$

$$F_n(x) = ©(x) + \tilde{A}(x) + R(x);$$

$\tilde{A}(x)$ $R(x)$ $»_1$ n $lim_{n\to\infty}R(x) =$
0 Berry $F_n(x)$ $©(x)$

$$|F_n(x) - ©(x)| \le A¯_3 n^{-\frac{1}{2}} \quad (\quad x);$$

$¯_3 = E|»_1|^3$ A (n $»_1$)
[22] Berry Cramér
$´ = \frac{1}{n}\sum_{i=1}^{n}(»_i - \bar{»})^2$ $\bar{»} = \frac{1}{n}\sum_{i=1}^{n} »_i$

$$G_n(x) = P(\sqrt{n}(´ - 1)/(\sqrt{®_4} - 1) \le x)$$

$(®_4 = E»_1^4)$ $®_6 = E»_1^6 < \infty$ $®_4 - 1 - ®_3^2 \ne 0$ $(®_3 = E»_1^3)$

$$|G_n(x) - ©(x)| \le \frac{A}{\sqrt{n}}\left(\frac{®_6}{®_4 - 1 - ®_3^2}\right)^{\frac{3}{2}} \quad (\quad x);$$

A $E»^{2k} < \infty$($k > 3$) $G_n(x)$

(6)

$\{»_n; n \ge 1\}$ $¹$
Robbins [23]
$" > 0$

$$\sum_{n=1}^{\infty} P(|\frac{1}{n}\sum_{k=1}^{n} »_k - ¹| \ge ") < \infty: \qquad (2)$$

$(\frac{1}{n}\sum_{k=1}^{n} »_k$ 1 $¹)$ (2) [23]
$\frac{1}{n}\sum_{k=1}^{n} »_k$ $¹$ \ "
[23]
(2) P. Erdös

(7)

Levy, Feller, Kolmogorov Gnedenko
[24] 1947
1968 Gnedenko Kolmogorov

Gnedenko( )

(8)

X $F(x)$

$$f(t) = \int_{-\infty}^{+\infty} e^{itx} dF(x):$$

[25] $f(t)$ $(-±; +±)($ $± > 0)$ X –

$$M_\beta(F) = \int_{-\infty}^{+\infty} |x|^\beta dF(x) < \infty$$

[26] $f(t)$ $(-\infty; +\infty)$
$(-±; +±)$
Gnedenko
$(-±; +±)$
$(-\infty; +\infty)$ $f(t)$ (U)
(U)

$F(x)$ $F(x)$ $p(x)$

$$p(x) = O[exp(-\frac{|x|}{\tilde{A}(|x|)})] \quad (|x| \to \infty);$$

$\tilde{A}(x)$

$$(lnx)^\lambda; \quad (lnx)(lnlnx)^\lambda; \quad ::: \quad (¸ > 1)$$

(9) [27]

$$A \to B = PA(P)^{-1};$$

P P P A B
A B [27]
( )
[28] A_1 A_2

$$A_1 \to B_1 = PA_1Q; \quad A_2 \to B_2 = PA_2Q;$$

P Q $(A_1; A_2)$ $(B_1; B_2)$

( )
[29] (38 54)
( ) A_1 A_2
$(A_1; A_2)$

$$A_1 \to B_1 = PA_1(P)^T; \quad A_2 \to B_2 = PA_2P^T;$$

P T $(A_1; A_2)$ $(B_1; B_2)$

(i) A_1 A_2 $(A_1; A_2)$
(7)
(ii) A_1 A_2
$(A_1; A_2)$ (8)

(10) [30] n
X $\mathcal{F}$ X Borel
$p(t; x; E)$ s x
$t + s$ E $(x \in X; t > 0; E \in \mathcal{F})$ [30]
$p(t; x; E)$ t $p(t; x; E)$ Austin
$p_{ij}(t)$

\ " ([31] [32])
[31] m (PBIB)
PBIB [32]
$X_1; :::; X_n$
$F(x)$ $»_1^{(n)} \leq ::: \leq »_n^{(n)}$ [32]
$»_{k_n}^{(n)}$ $(k_n \to \infty; k_n = n \to ¸ \in [0; 1))$ $»_{k_n}^{(n)}$

$$Á_1(x) = \frac{1}{\sqrt{2¼}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$$

$$Á_2(x) = \begin{cases} 0 & x \leq 0 \\ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha ln x + \beta} e^{-\frac{t^2}{2}} dt & x > 0; ® > 0 \end{cases}$$

$$Á_3(x) = \begin{cases} 1 & x \geq 0 \\ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha ln|x| + \beta} e^{-t^2} \end{cases}$$

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